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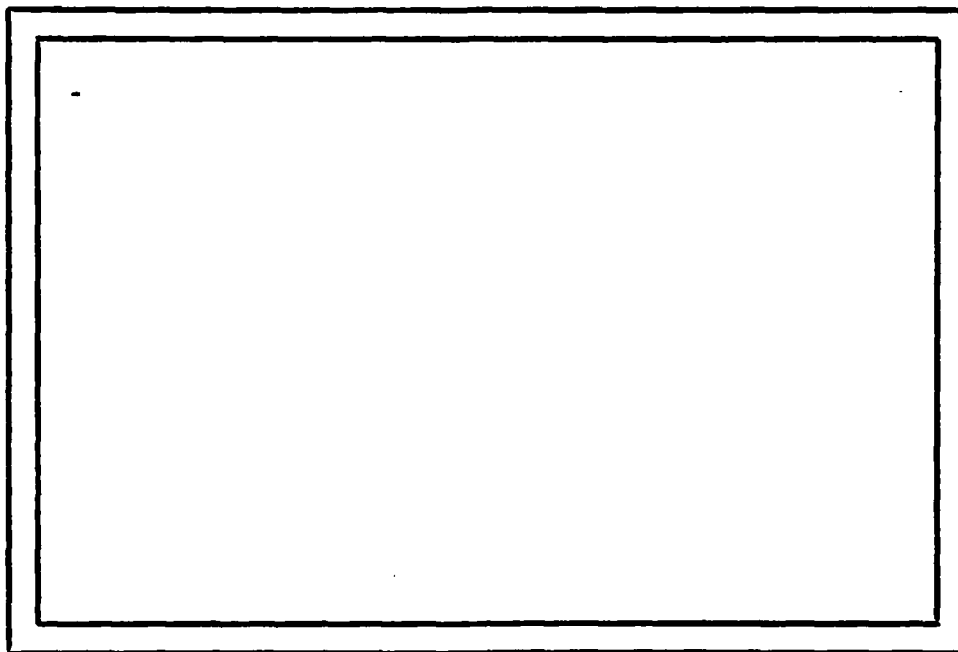
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TOWARD THE RECOGNITION
OF BUILDINGS AND ROADS
ON AERIAL PHOTOGRAPHS

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ABSTRACT

This paper describes steps toward the recognition of cultural features such as buildings and roads on aerial photographs. The approach involves several successive stages of grouping of edge segments. Straight line segments are fitted to sets of edge pixels; compatibilities between pairs of these segments, based on gray level and geometric information, are computed; and the segments are then grouped into building-like and road-like groupings based on these compatibilities. Examples of the results obtained using this approach are given, and some variations on the initial stages of the process are also investigated.

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1. Introduction

This paper describes an approach to the extraction of cultural features such as roads and buildings from aerial photographs. The approach involves three stages, at which successively more global knowledge about the features is used to guide the extraction process.

The approach taken in this paper was motivated by the following considerations:

a) It is necessary to develop methods that can deal with cases where map information, giving the approximate locations of the features to be extracted, is unavailable.

b) An effort has been made to use methods that can be implemented by parallel processing techniques, particularly at the lower levels. If inherently sequential methods, such as road tracking, are used too extensively, it will be difficult to implement the feature extraction process in real time.

c) In order to reduce computational costs, the approach has been broken up into stages, at which increasingly global and more specialized knowledge about the features to be extracted is used. The first stage involves local operations on pixels, using general information about the local properties (gray level, color, contrast, etc.) that pixels belonging to the features are likely to have. Since at this stage we are examining every pixel, it is important that only simple computations be performed. The principal output of this stage is a set of line segments representing fragments of feature edges, and labelled with various

property values computed for these fragments. The second stage groups these edge segments into pieces of features ("feature segments"), based on "semi-local" properties of the features (curvature, parallel-sidedness, etc.); the third stage groups the feature segments into global features, using global information about their shapes and spatial relationships. Thus at each stage, the computations are more complex, but they are applied to a smaller set of data.

d) Since the approach involves several successive stages of segmentation or grouping, if errors are made at an early stage, they may be difficult to correct at later stages. It is important to preserve the correspondences between entities at successive levels--i.e., between edge segments and the pixels that comprise them, and between feature segments and the edges of which they are composed; this will make it easier to locate the sources of any errors. It is also highly desirable to avoid firm decisions at any stage, and to avoid the use of processes that involve thresholds, but rather to make fuzzy or "probabilistic" decisions whenever possible, thus deferring commitments until they are confirmed by corroborating evidence. Note that when firm decisions are made, inputs that differ by arbitrarily small amounts may give rise to drastically different outputs. If such decisions must be made, they should be based on as much information as possible.

The successive stages in our approach are described in the

following sections of this paper, and specific motivations are given for the types of knowledge used at each stage.

2. Edge segments and groups: general concepts

2.1 Edge pixels

Cultural features often contrast with their surrounds, and are usually bounded by sharp, locally straight edges. These characteristics can be used as guidelines in classifying pixels as possibly belonging to such features. On the other hand, information about feature shapes and spatial relationships would normally not be very useful in making decisions about pixels, unless the information is very specific, i.e., template-like. Knowing that houses are rectangular, for example, does not help us in classifying a pixel as being possibly part of a house, so that we can say very little about how it should be related to other pixels if it is indeed part of a house.

If the features have characteristic gray levels or colors, we should certainly use these properties in making decisions at the pixel level; but in nonmultispectral imagery, it will usually not be possible to characterize features in this way. Moreover, if we do classify the pixels based on their gray levels, we will often obtain large connected components of constant gray level; thus using a very local classification criterion (the pixel's gray level) may give rise to relatively global segments, and this will often be unwarranted.

These considerations have led us to propose the use of an edge-based approach at the pixel level. We first use local operators to estimate the magnitude and direction of the gradient

at each point. We then use an iterative process at the pixel level to adjust the magnitudes and directions in the following way:

a) The magnitude is increased in the presence of high magnitudes at neighboring points in the tangential direction, provided their directions are smooth continuations of that direction; and it is decreased in the absence of such neighbors. This strengthens the edge responses at points that lie on straight or smoothly curved edges, and weakens them elsewhere.

b) At the same time, the direction is adjusted to make it agree more closely with these neighboring directions; the amount of adjustment depends on the magnitude at these neighbors. This tends to smooth out irregularities in the edge responses caused by noise.

c) An iterative scheme could also be used [1] for edge thinning: The magnitude is reduced in the presence of higher magnitudes at neighboring points in the gradient direction, and increased in the presence of lower magnitudes. If this is done iteratively, the magnitudes at the tops of the "ridges" of responses increase, while those at other points decrease, so that the edge responses are thinned.

Thus this process should produce sets of high-magnitude responses that lie on (thin) straight (or smoothly curved) edge segments, and such that the associated directions are locally very

consistent. Note that the process involves no thresholds or decisions, and that it is readily implementable in parallel.

Figure 1 illustrates the results of applying such processes to the edge responses in a small portion of an aerial photograph of the Occoquan, VA, area. The desired enhancement effects are all quite apparent. No thinning was done, so that the magnitude reinforcement process tends to thicken the edges; but this is not considered harmful, since in any case line segments will be fitted to the edges at the next step, and these will be much the same whether or not the edges are thin--in fact, they may be more reliable if the edges are thick. The specific algorithms used were described in an earlier technical report [2]. Many variations on these algorithms could have been used, and would have yielded similar results; e.g., see [3]. An edge enhancement relaxation scheme could also have been used [4].

2.2 Edge segments

We now want to construct a data representation based on
entities more global than pixels; this will allow us to use
more global knowledge about cultural features, e.g., simple types
of shape information, to classify these entities. Straight or
smoothly curved edge segments are obvious choices for these
entities, since the pixel-level processes tend to produce sets
of edge responses that lie along such segments.

Extracting edge segments inherently involves some sort of
threshold criterion, since one must decide whether or not to
construct a segment corresponding to a given collection of edge
responses. Such decisions should be easier for enhanced responses,
but they are still nontrivial, and should be made on the basis
of as much information as possible. If we simply threshold the
(enhanced) edge magnitudes, we are making the decisions on a pixel
by pixel basis, using only the information concerning that pixel,
which is undesirable. (Note, however, that when we do this for
enhanced responses, the information associated with a pixel also
reflects the nature of its neighbors.)

A somewhat safer idea is to make decisions about pixels in
the context of their neighborhoods. For example, one might "accept"
a pixel if its own magnitude, and the magnitudes of two of its
neighbors in the tangential directions, are sufficiently high.
(Note that this idea is very compatible with the enhancement
process; it essentially accepts just those pixels that would be

strongly enhanced.) At the same time, one can establish links between each accepted pixel and its neighbors; these links can then be used to define connected components of accepted pixels, which then constitute the desired edge segments. Such a linking approach is used by Navatia and Babu [5]. Alternatively, one can use a global straightness criterion in defining the connected components by requiring each pixel's direction to lie close to the average direction of the already accepted pixels [2]; this breaks up smooth curves into segments having relatively low net changes in slope from one end to the other. Figure 2 illustrates the types of edge segments obtained using this criterion.

It would be even more desirable to make decisions about entire groups of linkable dge pixels; but the number of such groups is enormous, and it is utterly impractical to consider all of them. However, suppose that we are only interested in groups of edge pixels that lie on a curve of a given shape, e.g., on a straight line. In this case we can use a Hough transform approach to map collinear sets of edge responses into compact peaks in the Hough space. We must then use a threshold criterion to detect the peaks, but this criterion is now being applied to an entire group of collinear edge pixels, rather than on a pixel by pixel basis. It should be mentioned that we obtain a cleaner Hough space when we use enhanced edge responses, since the slope estimates are much more consistent than in the raw responses, and this in turn makes the estimates of the distances of lines from the origin

much more consistent. Of course, we should not merely use
slope and distance (and response magnitude) to define clusters
in Hough space; other properties associated with the edge re-
sponses, e.g. the gray levels on the two sides of the edge,
should also be used if appropriate, to differentiate between
responses that (probably) belong to different edges. It may even
be desirable to use position along the line as a feature, in
order to avoid clustering responses that are far apart in the
image and have no responses between them. Such global approaches
to edge segment construction deserve further investigation.

2.3 Groups of segments

We now have a set of edge segments, with each of which we can associate various properties, including its length, average slope, average strength, etc., as well as properties of the gray levels on the two sides of the segment's constituent edge pixels, e.g., the means and standard deviations of these gray levels. If desired, we can now use this information to search for missing parts of edges in the original image, so as to fill gaps in the edge segments and create longer ones. We can also now group the edge segments into cultural feature segments, based on our knowledge about the expected geometrical properties of these segments. In this section we discuss some possible approaches to edge segment grouping. For simplicity, we consider two simple types of grouping, based, respectively, on good continuation and on parallelism.

Straight segments that are collinear, or curved segments that "point toward" one another, can be linked using criteria based on strength, length, distance, and good continuation, as well as similarity of properties [6]. (This assumes, of course, that such linking is consistent with what we know about the features that we are trying to extract.) Linking across large gaps can be done much more reliably at the segment level than at the pixel level, since the information that we have about the segments (slope, property similarity, etc.) is more reliable than the corresponding information about pixels. At the same

time, exploration of large gaps at the pixel level would involve
an excessive amount of computation per pixel.

This type of linking involves pairwise decisions; as pointed
out in Section 2.2., it would be preferable to make decisions about
entire groups of segments as to whether or not they constitute
good groupings, rather than making decisions about two segments
at a time. In general, it is not practical to consider all pos-
sible combinations of segments; but if we restrict ourselves to
sets of collinear segments (or more generally, segments that lie
on a curve of known shape), it is computationally feasible to
evaluate all possible sets of consecutive segments as possible
groupings. Various criteria for evaluating sets of collinear
segments have been formulated that yield perceptually reasonable
results [7]; Figure 3 illustrates one simple possibility.

In addition to segment linking based on collinearity or good
continuation, one usually also wants to link pairs of "anti-
parallel" segments, representing pairs of parallel edges whose
dark sides or light sides face one another, since cultural
features often have parallel sides. In the work of Nevatia and
Babu [5] and of Brooks [8], links are formed only for pairs
having no segments between them; but in general, we should be
allowed to link two segments even if there are other segments
between them, since these other segments may be due to noise, or
may represent features internal to the given one (e.g., a pent-
house on a building, a divider strip on a highway). Thus in

general we must compute link merits for many pairs of segments, and then choose "best" pairs for actual linking. The merit function may depend on the strengths, slopes, lengths, and property value similarity of the segments, as well as on their degree of overlap and on the distance between them, and on any special knowledge that we may have about the properties of the desired features. Note that the merit may be asymmetrical; for example, if a short segment and a long segment face one another, the merit of linking the short one to the long one may be much higher than that of linking the long one to the short one. Given the merits for all pairs of segments, we can link all pairs having (mutually) highest merit; once we have done this, the linked segments are no longer candidates for linking, so that some of the remaining pairs may now have mutually highest merit and can now be linked. This process can be repeated until no further linking is possible. Figure 4 shows the results of applying this process using a very simple merit function, namely the fraction by which one segment overlaps the other divided by the distance between them, provided the segments have approximately equal slopes. Several variations of this approach have also been tried, with essentially identical results [9]. An additional example is shown in Figures 1'-4', which are analogous to Figures 1-4.

The antiparallel linking schemes just described are all based on pairwise decisions. As before, it would be preferable

to evaluate groupings of segments that form antiparallel strips, rather than linking such segments two at a time. This would allow us to combine the collinear and antiparallel linking processes into a single strip clustering process. Here again, a Hough-like approach might be used to detect clusters arising from strips.

3. Edge segments: buildings and roads

Up to now we have discussed general approaches to the
problem of edge segment construction and grouping. In this
section we develop a more specialized approach, aimed at ex-
tracting groupings that represent buildings and roads on an
aerial photograph. Edge segments are constructed as described
in Section 2.2. We associate various properties with each seg-
ment, including its length, average edge strength, average
gray level on each side of it, etc. These properties are used
to derive initial probabilities that the segment is part of a
road, part of a building, or neither (we refer to this last
alternative as "other"). Groups of segments are then formed,
and the segment probabilities are updated based on properties
of the groups.

3.1 Average gray level calculation

In order to calculate the initial probability assignments, we have to find the average gray level on both sides of a line. The algorithm for calculation of average gray level on both sides of an edge segment is as follows:

1) Generate a strip of width "d" on each side of the segment. Find the co-ordinates of the points inside the two strips as well as the number of points on each side.

2) Calculate the average gray level on each side by dividing the sum of the gray levels by the number of points on each side.

The algorithm starts by reading in the coordinates of the end points of each line. Then the slope of the line is calculated. At this point it is determined whether the angle (θ) of the line with respect to the x-axis is between 0 and 90 degrees or is between 90 and 180 degrees. This differentiation is necessary in order to define a sense for each side of the line segment.

Referring to Figure 5, the end points are designated as end point 1 and end point 2. The sides are denoted similarly. Using the conventions in Figure 5, the following equations can be written for each edge segment and for the boundaries of the strips on both sides of each segment. When θ is not equal to 90 degrees we have:

$$y_0(x) = mx + mx_1 + y_1$$

$$y_{13}(x) = -x/m + x_1/m + y_1$$

$$y_{14}(x) = -x/m + x_2/m + y_2$$

$$y_{11}(x) = mx - m(x_1 + \Delta x) + y_1 - \Delta y$$

$$y_{12}(x) = mx - m(x_1 - \Delta x) + y_1 + \Delta y$$

where $\Delta x = d \sin\theta$, $m = (y_1 - y_2)/(x_1 - x_2)$

$\Delta y = d \cos\theta$ when $0 \le \theta < 90$

and $\Delta y = -d \cos\theta$ when $90 < \theta < 180$

When θ is equal to 90 degrees we have the following equations for the boundary lines of the strip. This case is shown in Figure 4 and the equations are:

$$x_0 = x_1 = x_2, \qquad x_{11} = x_0 + d$$

$$x_{12} = x_0 - d, \qquad y_{13} = y_1$$

$$y_{14} = y_2$$

The digitized image is given in the form of a rectangular matrix of elements $g(i,j)$ in which (i,j) are the Cartesian coordinates of a point and $g(i,j)$ is the value of the brightness at the point (i,j).

In order to calculate the gray level averages inside the strips, we sum up the gray levels of those points which satisfy the conditions below and divide by the number of points in the strip:

$$\text{Average gray level} = \sum_{i,j} g(i,j)/n$$

The points inside each strip should satisfy the following conditions:

1) When $0° \leq \theta < 90°$

    a) For side "1"

        $x_2 \leq i \leq x_1 + \Delta x$ $y_2 - \Delta y \leq j \leq y_1$

        $Y_{11}(i) < j < Y_0(i)$ $Y_{14}(i) < j < Y_{13}(i)$

    b) For side "2"

        $x_2 - \Delta_x \leq i \leq x_1$ $y_2(i) \leq j < y_1 + \Delta y$

        $Y_0(i) < j < Y_{12}(i)$ $Y_{14}(i) < j < Y_{13}(i)$

2) When $90° < \theta < 180°$

    a) For side "1"

        $x_1 \leq i \leq x_2 + \Delta x$ $y_2 \leq j \leq y_1 + \Delta y$

        $Y_0(i) < j < Y_{11}(i)$ $Y_{14}(i) < j < Y_{13}(i)$

    b) For side "2"

        $x_1 - \Delta x \leq i \leq x_2$ $y_2 - \Delta y \leq j \leq y_1$

        $Y_{12}(i) < j < Y_0(i)$ $Y_{14}(i) < j < Y_{13}(i)$

3) When $\theta = 90$

    a) For side "1"

        $x_1 - d \leq i < x_1$ $y_1 \leq j \leq y_2$

    b) For side "2"

        $x_1 < i < x_1 + d$ $y_1 \leq j \leq y_2$

3.2 Initial probability assignment

One of the most useful properties that can be used for calculation of the initial probability assignment vector is the average gray level in a strip on each side of the segment. These averages can then be compared with typical gray levels of cultural features such as roads or buildings. The minimum difference of these side average gray levels from the typical gray levels of roads and buildings is used as a figure of merit in the calculation of initial probabilities.

Roads and buildings are the brightest objects on the photographs that we used. They also have similar gray levels (similar reflectances) in the scene. Using these facts, in what follows an automatic method for estimating the gray level is described.

1) Calculate the average gray level in a strip on each side of each line segment.

2) Sort the line segments in decreasing order of length.

3) Select the longest p% of the lines (usually 5%).

4) Calculate the average gray levels of the brightest sides of the lines selected in step (3).

The average gray level calculated in this way can be accepted as a good estimate for the typical gray level of the objects.

To define the process of calculating the figures of merit more precisely, each line segment in the scene has two sides. The average gray levels of the strips along the two sides of the segment are denoted by g1 and g2 (see Figure 7).

Suppose that the typical average gray levels of roads and buildings are gr and gh respectively. Then the differences

$$f1 = |gr - g1| \quad \text{and} \quad f2 = |gr - g2|$$

measure the dissimilarity between the two sides of the line segment and the gray level of a typical road. Therefore, the function sr = min(f1,f2) is a measure of the gray level similarity between the given line segment and a typical road. Similarly the differences

$$h1 = |gh - g1| \quad \text{and} \quad h2 = |gh - g2|$$

measure the dissimilarity between the two sides of the line segment and the gray level of a typical building, and the function sh = min(h1,h2) is a measure of the gray level similarity between the given line segment and a typical building.

Finally,

$$s = min(sr,sh)$$

will be small if the gray level average on one of the sides of the line segment is close to the gray level of a typical building or road. Therefore, if s is small the line segment is more probable to be an edge of a road or a building than to be an "other" type of edge, whereas if s has a large value, the probability that the line segment is in the "other" class is high.

In order to express the value of s as a figure of merit, linear functions are used. Let di (i = 1,2,3) represent the figures of merit. To define them as linear functions of s, the following linear expression is used for calculation of a road

figure of merit. This linear function is shown in Figure 8 by thin solid lines.

$$d1 = \begin{cases} (1/dgr-1/gr)(g-gr) + 1 \\ \quad \text{when } (g^2r-2gr\cdot dgr)/(gr-dgr) \leq g \leq gr \\ 0 \quad \text{when } g^2r/(gr-dgr) < g < (g^2r-2gr\cdot dgr)/(gr-dgr) \\ (1/gr-1/dgr)(g-gr) + 1 \\ \quad \text{when } gr \leq g \leq g^2r/(gr-dgr) \end{cases}$$

Here dgr is the deviation allowed for road gray level; beyond it, the figure of merit of "other" will become greater than the figure of merit of road. The value of g is

$$g = g1 \quad \text{if } f1 < f2$$

and

$$g = g2 \quad \text{if } f1 > f2$$

Similarly the figure of merit for a line segment being a piece of building is shown by the thick solid lines in Figure 8 and its expression is as follows:

$$d2 = \begin{cases} (1/dgh-1/gh)(g-gh) + 1 \\ \quad \text{when } (g^2h-2gh\cdot dgh)/(gh-dgh) \leq g \leq gh \\ 0 \quad \text{when } g^2h/(gh-dgh) < g < (g^2h-2gh\cdot dgh)/(gh-dgh) \\ (1/gh-1/dgh)(g-gh) + 1 \\ \quad \text{when } gh \leq g \leq g^2h/(gh-dgh) \end{cases}$$

Here dgh is the deviation allowed for building gray level; beyond it, the figure of merit of "other" becomes greater than the figure of merit of buildings. The value of g is

$$g = g1 \quad \text{if } h1 < h2$$

and

$$g = g2 \quad \text{if } h1 > h2$$

When sr < sh road is more probable; therefore we use the dashed line for calculation of the figure of merit for "other". Similarly when sr > sh buildings are more probable and the dotted line is used for calculation of the figure of merit for "other". In summary, the figure of merit for the "other" class is calculated using the following formula:

When sr < sh

$$d3 = \begin{cases} |gr - g|/gr & \text{when } 0 < g < 2gr \\ 1 & \text{when } g \geq 2gr \end{cases}$$

Similarly when sr > sh

$$d3 = \begin{cases} |gh - g|/gh & \text{when } 0 < g < 2gh \\ 1 & \text{when } g \geq 2gh \end{cases}$$

The initial probability for each label is obtained by dividing the figure of merit of each label by the sum of the figures of merit of the three labels. Defining the initial probability in this manner, we have

$$p_\lambda^{(0)}(i) = di / \sum_{i=1}^{3} di \qquad i = 1,2,3$$

where di (i = 1,2,3) is the figure of merit of each label using the previous linear formulation and λ is the edge segment label.

When we use the functions in Figure 8, many segments will have probability 1 of belonging to the "other" class. These segments can be discarded as noise.

4. Pairs of segments: buildings and roads

The next step after noise cleaning is to group the line
segments in a meaningful manner. In order to do this, models
of the edges constituting objects should be used. The models
of roads and buildings used in the program will now be described.

a) The model of edges belonging to a piece of a road

From the function of a road, it follows that certain physical
and geometrical requirements must be satisfied. The properties
used in this model are as follows:

1) The spectral properties of a road correspond to materials
such as concrete and asphalt and it is usually homogeneous.

2) A piece of an edge of a road should have an anti-parallel
edge.

3) A piece of an edge of a road is usually connected to
other neighboring pieces with low angle deviation.

b) The model of edges belonging to a building

Similarly, the physical and geometrical properties of a
building are:

1) The spectral properties of the roof of the building.

2) The similarity of gray level inside the edges consti-
tuting a building.

3) A piece of an edge of a building is connected to other
pieces.

4) The edges of a building form a closed figure (usually
with right angles).

In order to use the above models the geometric relationships between each pair of lines within a neighborhood in the scene should be studied. In general, using the conventions of Figure 5, every pair of lines in the scene belongs to one of sixteen cases. These cases are listed in Table 1. The entry "side" in Table 1 refers to the object side of the given segment.

In order to find the object side of a line segment, first the two values sr and sh are calculated. Then, using the following decision rules the object side is found:

when sr < sh

   if f1 < f2   side = 1

else   side = 2

and

when sr > sh

   if h1 < h2   side = 1

else   side = 2

Assume that the pair of lines under study are labeled as line A and line B. The angles of the two lines with respect to the x-axis are θ_A and θ_B respectively. Depending on the orientation of the pair of lines, different angles between the two lines are possible. Figure 9 shows examples of the angle θ between two lines. The plus sign indicates the side of the road or building. According to this convention the angle between two collinear lines is 180°.

4.1 Compatible pairs

We now give the details of the algorithm for finding compatible pairs of segments, i.e., pairs that might be consecutive edge segments of a building or road. Referring to the model of edges constituting the objects, each of these pairs of lines should satisfy certain conditions in order to be accepted as a candidate compatible pair. In general, these conditions are:

a) Similarity of gray level of a strip along a line connecting their ends with respect to the object side of the pairs.

b) Conditions on the geometrical configuration of the pair of lines.

To check the similarity condition, the average gray level on the object side of the pair of lines is calculated by

$$g = (gA + gB)/2$$

where gA and gB are the average gray levels of the strips along the object sides of lines A and B. Then, the corresponding average gray level of a strip along a line connecting the ends of the lines is calculated. The difference between this value and g is a measure of the gray level similarity of the line connecting the two ends with the pair of lines. If this difference is within the limits used in calculation of the figures of merit, then the similarity condition is satisfied.

In a case where the distance between the ends is very small, that is, comparable with the width of the strip used in calculation of the gray level, the similarity measure is not reliable. This is because the number of points used in calculation of the gray level is limited. In cases where the distance between the ends of pair under study is less than the width of the strip used in calculation of the average gray level, the similarity condition will not be checked. In this case the pair is considered as a compatible candidate if the appropriate geometrical conditions are satisfied.

Geometrical conditions are important in making two lines compatible. Figure 10 and Figure 11 show examples of geometrically compatible and incompatible pairs, respectively. To differentiate between geometrically compatible and incompatible pairs, certain constraints on the geometrical locations of the end points are necessary. The ratio of the distances between end points can be used to reject the geometrically incompatible pairs.

In what follows, the first four cases in Table 1 will be analyzed and their compatibility conditions derived. The other cases have similar conditions.

Case (1)

Referring to Figure 12, there are five different configurations. In this case the compatibility of line A with respect to line B at end (2) or the compatibility of line B with respect to line A at end (1) is considered. Table 2 summarizes the conditions

imposed in these cases. The parameter m in the table is taken to be 1.5. This allows some overlap between the pairs of compatible line segments. The angle between the two lines is

$$\theta = \pi + |\theta_A - \theta_B| \qquad \text{if } \theta_B \leq \theta_A$$

and

$$\theta = \pi - |\theta_A - \theta_B| \qquad \text{if } \theta_B \geq \theta_A$$

Case (2)

In this case seven different configurations are considered. These are shown in Figure 13. The compatibility of end (1) of line A or line B is considered. Table 3 summarizes the required conditions. The angle between the two lines in this case is

$$\theta = 2\pi - |\theta_A - \theta_B| \qquad \text{when ya2} < y0$$

and

$$\theta = |\theta_A - \theta_B| \qquad \text{when ya2} > y0$$

where

$$y_0 = mb \cdot xa2 - mb \cdot xb1 + yb1$$

and mb is the slope of line B. The conditions at end (2) of the lines are similar to the end (1) conditions. To find these conditions a1 and b1 should be changed to a2 and b2 except that in this case

$$y_0 = mb \cdot xa1 - mb \cdot xb1 + yb1$$

The side similarity for some configurations is different in this case.

Case (3)

When the compatibility of end (1) of line A with end (2) of line B is considered, there are five different configurations. Figure 14 shows these configurations. The conditions are summarized in Table 4. The angle between the lines is

$$\theta = \pi + |\theta_A - \theta_B|.$$

The other possibility is to study the compatibility of end (2) of line A with end (1) of line B. Here again there are five different configurations. Figure 15 shows these configurations. The conditions are summarized in Table 5. The angle between the lines is

$$\theta = \pi - |\theta_A - \theta_B|.$$

Case (4)

The conditions for this case are summarized in Table 6 and Table 7. The different configurations are shown in Figure 16 and Figure 17. The angle between the lines is

$$\theta = |\theta_A - \theta_B|$$

when the compatibility of end (1) of line A is considered. Similarly the angle is

$$\theta = 2\pi - |\theta_A - \theta_B|$$

when the compatibility of end (2) of line A is in question.

So far the geometrical and similarity conditions for the pairs of compatible segments have been found. In what follows the algorithm for finding compatible pairs will be explained.

Algorithm for Finding Compatible Pairs

1) Choose those line segments whose "other" property is not equal to 1.

2) For end "1" of each line, find the shortest distances from other end points of line segments.

3) Find the object side of the given line and the other lines found in (2).

4) Check the geometrical and similarity conditions for the given line and the other lines found in (2). Reject those lines for which the required conditions are not satisfied.

5) If all the lines are rejected go to (8).

6) Find the angle of the line with respect to the remaining lines in (4). Choose the line which has the smallest angle (e.g. greater than 25°) with respect to the line under study.

7) Choose the other end of the line found in (6) and go on to (2).

8) Choose the other end of the given line and go to (2). If the other end has already been tested go to (9).

9) Continue the above process for the other line segments.

4.2 Antiparallel pairs

The edges of cultural features usually occur in pairs, as in the sides of roads and of buildings. To identify these features the edges should be clustered into antiparallel pairs (i.e. pairs of facing edges that are parallel but have opposite senses). Clustering must take into account information from the picture in the regions around the edges. For example, a road usually has a uniform gray level and thus it is reasonable to expect the facing sides of an antiparallel pair of edges to have similar gray levels.

The present method finds the pairs of lines that are antiparallel up to a certain angle difference (usually 25°) when similarity of gray level between the pairs is satisfied.

The basic procedure is as follows. A strip is moved along the object side of each side segment. The movement is continued until the similarity is lost or the distance moved is greater than the largest expected object size in the scene. While the strip moves, it hits other line segments. Among these line segments the following segments are rejected:

a) If they are not anti-parallel

b) If the difference in the angle is greater than a threshold.

The similarity is defined as the difference between the average gray level of the moving strip and the average gray level of the object side of the edge segment:

$|g-gmove|$ < level of similarity

where g = average gray level of the line segment

and gmove = average gray level of the moving strip.

The level of similarity used in the program is taken as 7, which is a rather tolerant condition. When the strip hits a candidate line the level of similarity is automatically changed to the value of the contrast of the candidate line. Note that this change of the level may stop the movement of the strip.

Among the remaining lines the one which has the smallest distance is selected as anti-parallel. To find the shortest distance between two anti-parallel line segments, at each end of the two segments perpendicular lines are drawn to the other line. If the intersection of the perpendicualr with the facing line is located outside of the line, the distance is neglected. Among the remaining distances, the minimum is selected as the distance between the two lines. Figure 18 shows an example of calculating the distance between two lines. As shown in this figure, among the four distances

d_i (i = 1,2,3,4)

d3 and d4 are rejected and

$d = \min(d_1, d_2)$

is selected as the distance between the two lines.

To check whether the intersection of the perpendicular line is between the end points of a line the following decision rules are used:

if θ is not equal to 90 degrees

and x1 ≤ xint ≤ x2 the intersect point is between the end points

else if θ is equal to 90 degrees

and y1 ≤ xint ≤ y2 the intersect point is between the end points.
Here (xint,yint) are the coordinates of the intersection point.

This method of finding anti-parallel pairs of lines has the following advantages:

a)   Each line is not compared with all other lines.

b)   When several lines are facing a line, the method allows all of these lines to choose the same line as anti-parallel.

c)   The method uses the context of the lines on the picture, namely, the similarity of the gray levels inside the object.

In what follows the algorithm for finding anti-parallel pairs is explained.

Algorithm for finding anti-parallel pairs

1)   Choose a line segment and find its object side.

2)   Generate the strip (a width of 4 points is used), and find the average gray level inside the strip.

3)   Move this strip parallel to the segment. While the similarity and the total movement distance are less than the specified levels, note the lines hit by the strip. If no lines are found go to (5). Otherwise, reject those lines where the angle difference is greater than the specified threshold and the facing side is not opposite to the original line. Set the similarity level equal to

the contrast of the line found and continue the process.

4) For the candidate lines found in (3) choose the one which has the minimum distance. Mark the line found in order not to process it again.

5) Continue the process for the remaining lines.

The maximum moving distance in the above algorithm is quite relaxed; it is set to be equal to be 1/4 of the size of the picture. For scenes containing small objects this distance can be reduced in order to reduce computation time. The angle difference can be set arbitrarily. The program is not sensitive to this threshold since the strip moves along the object side of the edge and so it is expected that we get another side of the object as the best candidate.

5. Groups of segments: buildings and roads

After application of the programs described up to now, we
have groups of compatible and antiparallel pairs of segments.
Using the model of roads and buildings, we want to update the
probabilities that were initially obtained using gray level
information. Based on these probabilities we can recognize
objects with good confidence or fair confidence.

We begin by dividing the groups of compatible pairs into
the following categories:

A)   Closed groups

B)   Semiclosed groups

C)   Other lines and groups

In what follows each of the above categories will be explained
in more detail.

A)   Closed groups

By a closed group, we mean that the start and the end segment
labels are the same. Figure 19 shows an example of this type
of group. In this figure A,B,C.... are the labels in a compati-
ble group.

It is obvious that this kind of closed group is a good can-
didate for being the group of edges of a building. To check
whether this closed group is a building, we test for solidness
inside the object sides, and also check that each line segment
in the group is antiparallel to a line in the group. To check
solidness we use the same operator that was used in finding the

antiparallel pairs. This test also guarantees the similarity
of gray level inside the object.

The above check can differentiate between the cases (b) and
(c) in Figure 19. Thus a closed group with the above conditions
can be considered a building with good confidence.

B) Semiclosed groups

A semiclosed group is defined as a group with a gap less than
the longest line connecting the ends of compatible pairs in the
group. Figure 20 demonstrates an example of this type. As in
the case of a closed group, if the following tests are valid,
then the group is accepted as a building with good confidence.

1) Solidness

2) Each line should be antiparallel to a line in the group.

Operators similar to those used for checking closed groups
are used here.

C) Other lines and groups

Here again the model of the edges constituting a building or
road will be used for the recognition of the remaining lines or
groups. The important features are the angles between the com-
patible pairs and information on anti-parallel pairs. Figure 21
shows examples of possible cases that may occur in the scene. In
this figure θmin is around 200°. Special care should be taken
in cases where the anti-parallel pairs or compatible pairs are not
available, due to cutoff at an edge of the frame.

We now describe in detail the algorithm for updating the probabilities of "other" lines or groups. A reinforcement algorithm is employed to update the probabilities of the remaining line segments by rewarding and punishing (increasing or decreasing a component of the probability vector). Here again the model of the edges constituting the objects will be used for the updating process. The most important features are the angles between compatible pairs and information on antiparallel pairs. For example, two anti-parallel lines should reinforce each other for both buildings and roads.

We begin by dividing the "other" lines and groups into the following categories.

1) Groups consisting of two compatible lines

2) Groups consisting of more than two compatible lines

3) Single lines

In what follows the criteria for classification of each of the above categories will be explained in more detail.

1) Groups consisting of two compatible lines

The two compatible lines are called A and B. There are four cases.

Case a

Both A and B have no anti-parallel lines due to cutoff at the edges of the frame. Figure 22 shows examples of this case.

If the angle θ between the lines is close to 90°, there is a high probability that the lines are a part of a building. If the

angle θ is greater than 90° the probability that the edge is a part of a road is higher. Similarly, if the angle is less than 90° the probability of being "other" is higher. In order to express this situation the following figures of merit are defined:

$$d1 = f1(\theta) \ [P_A^{(0)}(1) + P_B^{(0)}(1)]$$

$$d2 = f2(\theta) \ [P_A^{(0)}(2) + P_B^{(0)}(2)]$$

$$d3 = f3(\theta) \ [P_A^{(0)}(3) + P_B^{(0)}(3)]$$

where d1, d2, and d3 are the figures of merit for road, building, and other, respectively. The functions $fi(\theta)$ $(i = 1,2,3)$ are defined as follows:

if $0 \leq \theta \leq \pi/4$

$$f1(\theta) = 0, \ f2(\theta) = 0, \ f3(\theta) = 1$$

if $\pi/4 < \theta \leq \pi/2$

$$f1(\theta) = 0, \ f2(\theta) = 1-|\cos\theta|, \ f3(\theta) = 0.25$$

if $\pi/2 < \theta \leq \pi$

$$f1(\theta) = |\cos\theta|, \ f2(\theta) = 1-|\cos\theta|, \ f3(\theta) = 0.25$$

if $\pi < \theta < 2\pi$

$$f1(\theta) = 0.5, \ f2(\theta) = 0, \ f3(\theta) = 0.5$$

Case b

One of the lines has no anti-parallel due to cutoff at an edge of the frame, and the other line has an anti-parallel line. Figure 23 shows examples of this case.

In this case again the figures of merit are defined as

$$d1 = f1(\theta) \, [P_{A'}^{(0)}(1) + 2P_B^{(0)}(1) + P_A^{(0)}(1)]$$

$$d2 = f2(\theta) \, [P_{A'}^{(0)}(2) + 2P_B^{(0)}(2) + P_A^{(0)}(2)]$$

$$d3 = f3(\theta) \, [P_{A'}^{(0)}(3) + 2P_B^{(0)}(3) + P_A^{(0)}(3)]$$

where label A' is the label of the anti-parallel line of A.
The functions $fi(\theta)$ $(i = 1,2,3)$ are the same as before. In this
definition the probability components of line B are counted
twice. This is because line B reinforces both lines A' and A.

For these two cases the update probability is defined as

$$P_\lambda^{(1)}(i) = di / \sum_{i=1}^{3} di \qquad (i = 1,2,3)$$

where λ is the label of the line under study.

Case c

Both A and B have anti-parallel lines. Figure 24 shows
possible examples of this case. In order to recognize this
case lines are drawn between the midpoint of each line and the
corresponding anti-parallel line. If the intersection of these
two lines is located between the lines, we have a building with
good confidence and therefore:

$$P_\lambda^{(1)}(1) = 0, \qquad P_\lambda^{(1)}(2) = 1, \qquad P_\lambda^{(1)}(3) = 0$$

for $\lambda = A,B,A',B'$.

If the intersection is outside the lines we have a road with good
confidence:

$$P_\lambda^{(1)}(1) = 1, \qquad P_\lambda^{(1)}(2) = 0, \qquad P_\lambda^{(1)}(3) = 0$$

for $\lambda = A,B,A',B'$.

Case d

This case includes all situations not covered by cases
(a-c)--e.g., both A and B have no anti-parallel lines and this
is not due to cutoff. In these cases no change is made to the
probabilities.

In the above four cases if the anti-parallel line of A or B
was previously recognized as a part of a road or as a part of a
building, A or B is considered as a line with no anti-parallel.
The reason for this is that the anti-parallel line was previously
recognized as a part of an object and therefore it should not
reinforce these lines.

2) Groups consisting of more than two compatible lines

In this case a line may have two compatible lines at its ends.
Examples of possible roads in this case are shown in Figure 25
and an example of a possible building is shown in Figure 26.
Notice that the example of a possible building shows the situation after the recognition of closed and semi-closed groups.

A piece of an edge segment A is classified as road with good
confidence if the following conditions are satisfied:

   a) If it possesses an antiparallel line A'

   b) If $\theta_1 > \theta min$ and $\theta_2 > \theta min$

   c) If $\theta_1' > \theta min$ and $\theta_2' > \theta min$

or if the line has no antiparallel because of cutoff but the
neighboring compatible line satisfies the above conditions.

Since we do not allow sharp turns for roads θmin is set to be 110°.

Special care should be taken with the start and end lines of a group where the condition for one angle, θ_1 or θ_2 is satisfied. The other special case is when one or both of the compatible lines have no antiparallel because of cutoff. In this case the condition on θ_1' or θ_2' will not be checked.

There are other cases where a line does not have an anti-parallel line but not because of cutoff. An example of this is shown in Figure 27. In this case if both compatible neighbors at the ends are classified as road with good confidence, then this piece will also be classified as road with good confidence. For all the above cases

$$P_\lambda^{(1)}(1) = 1, \qquad P_\lambda^{(1)}(2) = 0, \qquad P_\lambda^{(1)}(3) = 0$$

for $\lambda = A, A'$.

An edge segment is classified as a road with low confidence if it satisfies the following conditions:

a) It has no anti-parallel, not due to cutoff

b) Only one compatible neighbor is a road with good confidence.

In this case the probability is updated as follows:

$$P_A^{(1)}(1) = 0.5, \qquad P_A^{(1)}(2) = 0, \qquad P_A^{(1)}(3) = 0.5$$

To classify cases where θ_1 or θ_2 is less than θmin, the following procedure is applied:

a) Within the group, we start from the line with angle less than θmin; then other neighboring lines are found that satisfy the same angle condition. Now we have a semi-closed group candidate. If the tests stated for semi-closed groups are valid, then we classify these line segments as buildings with good confidence.

b) If the semi-closed tests are not valid, the classification is done according to the category of groups consisting of two compatible lines. Here the lines are considered two by two and the classification is done as before.

The recognition for other cases (e.g., none of the lines have antiparallels) is done according to the category of groups consisting of two lines as before.

3) Single lines

These are single lines with no compatible lines at the ends. The following rules are applied for classification of single lines.

a) Classify a single line as road if it has an anti-parallel line.

b) Classify it as "other" if it has no anti-parallel.

A simple verification step for isolated road lines is added in order to reject noisy lines that have prematurely been recognized as road with good confidence. The isolated road lines are verified if they have a minimum acceptable length for roads.

6. Examples

The algorithms described in Sections 3-5 were applied to three images showing portions of a suburban area near Occoquan, VA (compare Figs. 1'-4').

Figure 28 shows one of the images, and Figure 29 shows the lines extracted from it. The average gray levels were calculated using strips of width 4, based on knowledge about the resolution of the image. The typical gray levels of roads and buildings were taken to be equal in calculating the probability vectors. The histogram of "other" probabilities is shown in Figure 30. From this histogram it is clear that we can completely differentiate between two classes of object boundaries, namely objects and noise. Figure 31 shows the line segments whose probabilities of being a piece of road or building are not equal to zero. This figure shows quite an improvement in rejecting the noise edges. Figure 32 shows the results of finding compatible segments, and Figure 33 shows the results of finding anti-parallel segments: the midpoints of the anti-parallel pairs are connected together. Figures 34 and 35 show the high confidence buildings and roads; Figures 36 and 37 show the buildings and roads with probabilities ≥ 0.75, and Figures 38 and 39 show those with probabilities ≥ 0.5.

Two other examples are shown in Figures 40 - 50 and 51- 61; these are analogous to Figures 28 - 39, except that the histograms of "other" probabilities are not shown. Two further examples

taken from the same aerial photograph, but involving non-residential buildings, are shown in Figures 62-69 and 70-77.

7. Variations

7.1 Edge segment adjustment

The bottom-up nature of the approach described in Sections 3-5 makes the results dependent on good choices of the initial edge segments. In this section several experiments are described aimed at improving the positions, orientations, or lengths of these segments.

a) Changing the segment's position

In this experiment the line segments were moved a few steps in each direction so that they remained parallel with the given line. Two different figures of merit, namely the maximum gradient and minimum standard deviation of the gray level on both sides of each line, were used to evaluate the position of a line segment. Referring to Figure 7ℓ, at each step the new end coordinates are

$$x11 = x1 + \Delta x \qquad x21 = x2 + \Delta x$$
$$y11 = y1 - \Delta y \qquad y21 = y2 - \Delta y$$

for side 1 and

$$x12 = x1 - \Delta x \qquad x22 = x2 - \Delta x$$
$$y12 = y1 + \Delta y \qquad y22 = y2 + \Delta y$$

for side 2 where $\Delta x = d_i \sin\theta$

$$\Delta y = d_i \cos\theta \quad \text{when} \quad 0 \leq \theta < 90$$

and $\quad \Delta y = -d_i \cos\theta \quad$ when $90 \leq \theta < 180$

where d_i is the distance moved at step i and θ is the angle of the line with respect to the x-axis.

At each step the average gray levels on side 1 and side 2 are calculated. The gradient at step i is

$$gi = g2-g1$$

where g1 and g2 are the average gray levels on side 1 and side 2. The maximum gradient is calculated as

$$gmax = max|gi|$$

The end coordinates of the line associated with the position of the maximum gradient are selected as the new coordinates of the line segment.

Figure 79 shows the effects of this method on the line segments of Figure 29 for four steps of movement in each direction and di = 1,2,3,4. The result of this experiment is that no major change of position was made for long line segments, but there were some bad effects on the short line segments. The reason is that short line segments are moved towards the neighboring long segments where the gradient is maximum.

Another figure of merit, the minimum standard deviation of the gray levels along the line segments, was also used to relocate the lines. The standard deviation at step i is

$$\sigma_i^2 = \sum_{k,j} (g(k,j)-g)$$

where $g(k,j)$ is the gray level of point (k,j) and g is the average gray level inside each strip. The minimum standard deviation is calculated as

$$\sigma_i min = min|\sigma_i|$$

Figure 80 shows the effects of this experiment. As shown in
this figure the line segments have the tendency to relocate
themselves where the gray levels are more uniform. The result
is that they usually move towards the center of the objects
and therefore more confusion will occur.

b) Changing the segment's angle

In this experiment each line segment is rotated around its
center point in both directions, a few degrees at each step.
As before, two different figures of merit were used to relocate
the line segments. Referring to Figure 81, at each step the new
coordinates are

$$x_1' = x_1 - \Delta x \qquad\qquad x_2' = x_2 + \Delta x$$
$$y_1' = y_1 + \Delta y \qquad\qquad y_2' = y_2 - \Delta y$$

where $\quad \Delta x = \ell \sin \psi_i/2 \sin(\theta + \psi i/2)$

$$\Delta y = \ell \sin \psi_i/2 \cos(\theta + \psi i/2)$$

where ψ_i is the angle of rotation at each step and ℓ is the
length of the line. In this formulation if the line is rotated
anti-clockwise ψ_i is taken to be a positive number; otherwise
φ_i is taken to be a negative number.

Figure 82 shows the results of changing the angles after
relocating the lines in the position of maximum gradient. In
general no improvement has been made regarding the positions of
the line segments in the scene. Similarly, Figure 83 shows the
result of changing the angles and relocating the lines in the

position of minimum standard deviation. Here again no improvement has been made.

In the above experiments φ_i is taken to be $i \times 5°$ and four steps of rotation are allowed. The width of the strip along the line segment is taken to be 4. Other experiments such as first rotation and then translation or vice versa have been performed and similar results have been found. The results are shown in Figures 84 to 87.

c) Changing the segment's length

When lines are fitted to components of edge points, in some places the lines overshoot the components at their ends. This may be due to noise in the edges near their ends or to the presence of nearby edges with similar directions. To study this effect some experiments have been performed to adjust the length of the fitted lines. The similarity of gray scale along the object side of the segment is used for this type of adjustment. Referring to Figure 88, the average gray level along the object side of the line (here the brighter side) is calculated. Then the average gray level along the object side near each end is calculated. A square of 4*4 is used for this purpose. Let g be the average gray level along the object side and g_{end} be the average gray level in the small neighborhood near the end. If the difference

$$|g - g_{end}| > \text{Threshold}$$

the length is reduced. The new coordinates are

$$x11=x1-d\cos\theta \qquad x22=x2+d\cos\theta$$

$$y11=y1-d\sin\theta \qquad y22=y2+d\sin\theta$$

where d is the width of the strip for calculation of average
gray level. This process is continued for both ends until

$$|g-g_{end}| < \text{Threshold}$$

When this inequality is satisfied the corresponding end coordinates are selected as the new end coordinates for the line segments.

The results of this experiment for different thresholds (5,4,3) are shown in Figures 89-91. When the threshold is high (5) only a few changes occur in the lines. When the threshold is 3 or 4 both the overshot lines and some of the other lines become shorter. This shows that a unique threshold cannot be used for all the lines. The best result is obtained when the threshold is dynamically set equal to the standard deviation of the gray level of the object side for each line segment. The result is shown in Figure 92.

From the experiments described in (a-c) above it appears that no improvement results from adjustment of the positions or orientations of the line segments; but there is some improvement when the lengths are adjusted using a dynamic thresholding process as described above. However, these improvements have little

effect on the final results. For example, the building and
road segments with confidences of 1, 0.75, and 0.5 obtained
after maximum-gradient angle adjustment are shown in Figures
93-98. Analogous results for dynamic-threshold length adjust-
ment are shown in Figures 99-104.

7.2 Shadow detection

One of the features that can be used for verification of
the recognition of buildings is the shadow of the building.
There are cases where a parking lot can be recognized as a
building, since they may have the same size or shape. It seems
that it is possible to use the shadow for further verification.
To study this, some experiments have been performed using the
average gray level within a strip along each line segment and
the angle of the line segment.

Referring to Figure 105, each line segment with angle θ with
respect to the x-axis has two sides. There are two average
gray levels associated with each line segment. The average
gray levels g1 and g2 are associated with angles θ and $2\pi-\theta$
respectively.

Scatter plots of g1 and g2 with respect to θ are shown in
Figure 106 for all of the line segments and in Figure 107 for the
line segments after noise cleaning. Figure 107 shows that at
dark gray levels, the population of points for $\theta > 180°$ is greater
than the population of points for $\theta < 180°$. This shows that in
certain orientations along the line segments, there exist dark
shadow regions.

To study this effect quantitatively, let us pick the darkest
P% of the population. Let

n1 = number of dark points in the interval $(\theta, \theta+\pi)$
and
n2 = number of dark points in the interval $(\theta+\pi, \theta+2\pi)$

for θ = 0,20,40,....,340. The plots of n1/n2 as a function of

θ for different values of P and for the line segments after

and before noise cleaning are shown in Figure 108 and Figure 109

respectively. These figures show that there is a peak around

180°. The peak is greater for the segments after noise removal.

As expected, if P is increased the peak becomes smaller. This

effect was tested on several other scenes and similar results

were obtained.

These results show that shadow detection is most reliable

after noise segments have been eliminated. Shadows could be

used to verify the recognition of buildings (which may have

shadows) and roads (which should not).

8. Concluding remarks

The approach used in this paper is quite elementary and straightforward. It proceeds in an essentially bottom-up fashion, with no provision, as yet, for top-down feedback between levels, and it makes no use of higher-level information, e.g. that buildings are alongside roads, or that roads form a connected network. It uses less knowledge than the road-finding systems of Fischler et al. [10], and handles fewer types of objects than the aerial photographic interpretation system of Nagao et al. [11]. Nevertheless, it serves to illustrate the level of performance that can be achieved by a straightforward hierarchical system. It is expected that this performance will continue to improve as additional levels of knowledge, and a more flexible control structure, are incorporated into the system.

It would be of interest to investigate a relaxation-like (or MSYS-like) scheme for classifying the feature segments. Initially, each individual segment would be probabilistically classified, on the basis of its properties, as being (part of) a road, building, etc. These probabilities would then be adjusted based on their compatibilities with those of nearby or otherwise related segments. One should not expect that a simple algebraic formula can be used to compute the probability adjustments; rather, they would be computed by a probabilistic "decision tree" associated with each segment. This approach should result in a generally consistent classification (which, of course, may

still be ambiguous). If inconsistencies remain, they would probably reflect errors in the feature segment extraction process, assuming that the compatibility models are adequate.

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Case	Geometrical conditions	Similarity conditions for the line a_2b_1
1	$a_2b_1 < a_2b_2/m$ $a_2b_1 < a_1b_1/m$	If $a_2b_1 < d$ or $x_pb_1 < d$ no check or $x_pa_2 < d$
(a)	$\theta_A \neq \theta_B$ $x_{a_2} \leq x_p < x_{a_1}$ $x_{b_2} < x_p \leq x_{b_1}$	no check
(b)	$x_{a_2} < x_{b_1}$ $y_{a_2} \leq y_{b_1}$	check side 2
(c)	$x_{a_2} > x_{b_1}$ $y_{a_2} \geq y_{b_1}$	check side 1
(d)	$x_{a_2} < x_{b_1}$ $y_{a_2} > y_{b_1}$	check side 1
(e)	$x_{a_2} \geq x_{b_1}$ $y_{a_2} < y_{b_1}$	check side 2

Table 2. Geometrical and similarity conditions for case 1.

# of cases	Segment A case	Segment A side	Segment B case	Segment B side	Similar to:
1	a	1	a	1	--
2	a	1	a	2	--
3	a	1	b	1	--
4	a	1	b	2	--
5	a	2	a	1	#2
6	a	2	a	2	#1
7	a	2	b	1	#4 rotated 180°
8	a	2	b	2	#3 rotated 180°
9	b	1	a	1	#3
10	b	1	a	2	#7
11	b	1	b	1	#1
12	b	1	b	2	#2 rotated 90°
13	b	2	a	1	#4
14	b	2	a	2	#8
15	b	2	b	1	#12
16	b	2	b	2	#6

Table 1. Different cases of pairs of line segments according to the conventions of Figure 5.

	Geometrical conditions	Similarity conditions for line a_1b_1
Case 2	$a_1b_1 < a_2b_2/m$ $a_1b_1 < a_1b_2/m$ $a_1b_1 < a_2b_1/m$	If $a_1b_1 < d$ no check
(a)	$\theta_A \neq \theta_B$ $x_{a_2} < x_p \leq x_{a_1}$ $x_{b_2} < x_p \leq x_{b_1}$ $x_pa_1 < x_pa_2/m$, $x_pb_1 < x_pb_2/m$	no check
(b)	$y_{a_2} < y_0$ $x_{a_1} < x_{b_1}, y_{b_1} > y_{a_1}$	If $\theta_A \neq \theta_B$ and $x_pa_1 < d$ no check else check side 1
(c)	$y_{a_2} < y_0$ $x_{a_1} > x_{b_1}, y_{b_1} \leq y_{a_1}$	If $\theta_A \neq \theta_B$ and $x_pb_1 < d$ no check else check side 2
(d)	$y_{a_2} < y_0$ $x_{a_1} \geq x_{b_1}, y_{b_1} > y_{a_1}$	check side 1
(e)	$y_{a_2} > y_0$ $x_{a_1} > x_{b_1}, y_{a_1} > y_{b_1}$	If $\theta_A \neq \theta_B$ and $x_pb_1 < d$ no check else check side 2
(f)	$y_{a_2} > y_0$ $x_{a_1} < x_{b_1}, y_{a_1} \leq y_{b_1}$	If $\theta_A \neq \theta_B$ and $x_pa_1 < d$ no check else check side 1
(g)	$y_{a_2} > y_0$ $x_{a_1} \leq x_{b_1}, y_{a_1} > y_{b_1}$	check side 2

Table 3. Geometrical and similarity conditions for Case 2.

	Geometrical conditions	Similarity conditions for line a_1b_2
Case 3 end 1	$a_1b_2 < a_2b_1/m$	If $a_1b_2 < d$
	$a_1b_2 < a_2b_2/m$	no check
	$a_1b_2 < a_1b_1/m$	
	$x_{a_2} < x_p \leq x_{a_1}$	
	$x_pa_1 < x_pa_2/m$	
(a)	$x_pb_2 < x_pb_1/m$	no check
	If $\theta_B \neq 90°$	
	$x_{b_1} < x_p \leq x_{b_2}$	
	else $y_{b_2} < y_p < y_{b_1}$	
(b)	$x_{a_1} \geq x_{b_2}$	check side 1
	$y_{a_1} < y_{b_2}$	
(c)	$x_{a_1} < x_{b_2}$	check side 1
	$y_{a_1} \leq y_{b_2}$	
(d)	$x_{a_1} < x_{b_2}$	If $x_pa_1 < d$ no check
	$y_{a_1} > y_{b_2}$	else check side 2
(e)	$x_{a_1} > x_{b_2}$	If $x_pb_2 < d$ no check
	$y_{a_1} \geq y_{b_2}$	else check side 2

Table 4. Geometrical and similarity conditions
for case 3: end 1 of line A.

	Geometrical conditions	Similarity conditions for line a_2b_1
Case 3 end 2	$a_2b_1 < a_1b_2/m$	If $a_2b_1 < d$
	$a_2b_1 < a_2b_2/m$	no check
	$a_2b_1 < a_1b_1/m$	
	$x_pa_2 < x_pa_1/m$	
(a)	$x_pb_1 < x_pb_2/m$	no check
	$x_{a_2} \leq x_p < x_{a_1}$	
	If $\theta_B \neq 90°$	
	$x_{b_1} \leq x_p < x_{b_2}$	
	else $y_{b_2} < y_p \leq y_{b_1}$	
(b)	$y_{b_1} \leq y_{a_2}$	If $x_pa_2 < d$ no check
	$x_{a_2} > x_{b_1}$	else check side 1
(c)	$y_{b_1} < y_{a_2}$	check side 1
	$x_{a_2} \leq x_{b_1}$	
(d)	$y_{a_2} \leq y_{b_1}$	If $x_pb_1 < d$ no check
	$x_{a_2} < x_{b_1}$	else check side 2
(e)	$y_{a_2} \leq y_{b_1}$	If $x_pa_2 < d$ no check
	$x_{a_2} > x_{b_1}$	else check side 2

Table 5. Geometrical and similarity conditions for case 3: end 2 of line A.

	Geometrical conditions	Similarity conditions for line a_1b_1
Case 4 end 1	$a_1b_1 < a_2b_2/m$	If $a_1b_1 < d$ no check
	$a_1b_1 < a_2b_1/m$	
	$a_1b_1 < a_1b_2/m$	
	$x_pa_1 < x_pa_2/m$	
	$x_pb_1 < x_pb_2/m$	
(a)	$x_{a_2} < x_p \leq x_{a_1}$	no check
	If $\theta_B \neq 90°$	
	$x_{b_1} \leq x_p < x_{b_2}$	
	else $y_{b_2} < y_p < y_{b_1}$	
(b)	$x_{a_1} \doteq x_{b_1}$	check side 2
	$y_{a_1} > y_{b_1}$	
(c)	$x_{a_1} < x_{b_1}$	check side 1
	$y_{a_1} \leq y_{b_1}$	
(d)	$x_{a_1} > x_{b_1}$	If $x_pb_1 < d$ no check
	$y_{a_1} > y_{b_1}$	else check side 2
(e)	$y_{a_1} < y_{b_1}$	If $x_pa_1 < d$ no check
	$x_{a_1} \geq x_{b_1}$	else check side 1

Table 6. Geometrical and similarity conditions for Case 4: end 1 of line A.

	Geometrical conditions	Similarity conditions for line a_2b_2
Case 4 end 2	$a_2b_2 < a_1b_1/m$	If $a_2b_2 < d$ no check
	$a_2b_2 < a_2b_1/m$	
	$a_2b_2 < a_1b_2/m$	
	$x_pa_2 < x_pa_1/m$	
	$x_pb_2 < x_pb_1/m$	
(a)	$x_pa_2 \leqslant x_p < x_{a_1}$	no check
	If $\theta_B \neq 90°$	
	$x_{b_1} < x_p \leqslant x_{b_2}$	
	else $y_{b_1} \leqslant y_p < y_{b_2}$	
(b)	$x_{a_2} > x_{b_2}$	If $x_pa_2 < d$ no check
	$y_{a_2} \geq y_{b_2}$	else check side 1
(c)	$x_{a_2} \leqslant x_{b_2}$	If $x_pa_2 < d$ no check
	$y_{a_2} > y_{b_2}$	else check side 1
(d)	$x_{a_2} \geq x_{b_2}$	If $x_pb_2 < d$ no check
	$y_{a_2} < y_{b_2}$	else check side 2
(e)	$x_{a_2} < x_{b_2}$	If $x_pb_2 < d$ no check
	$y_{a_2} < y_{b_2}$	else check side 2

Table 7. Geometrical and similarity conditions for Case 4: end 2 of line A.

Figure 1. (a) Window of the Occoquan photograph, showing parts of Lorton reformatory
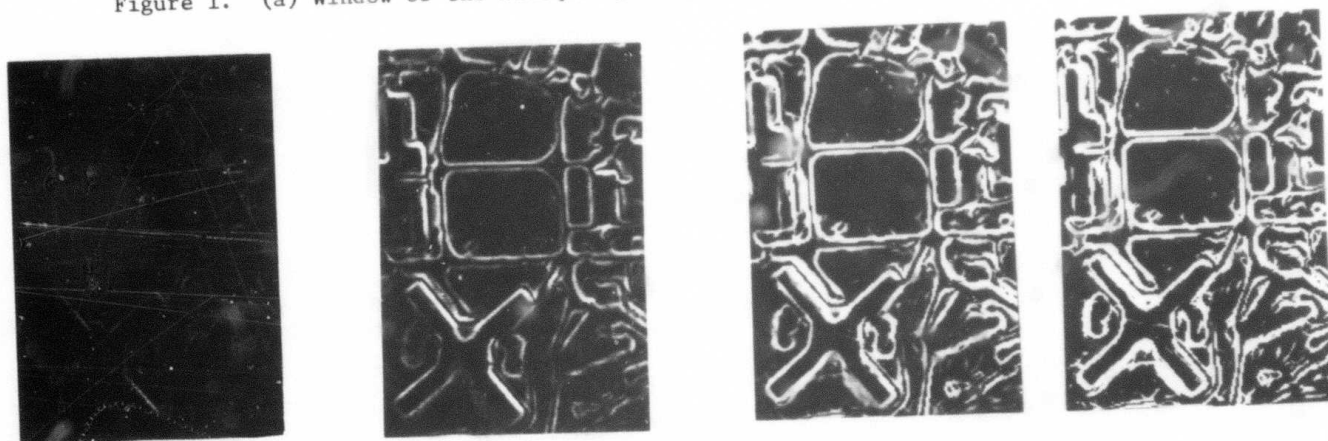
Figure 1. (b) Original Sobel gradient magnitudes and three iterations of the enhancement process
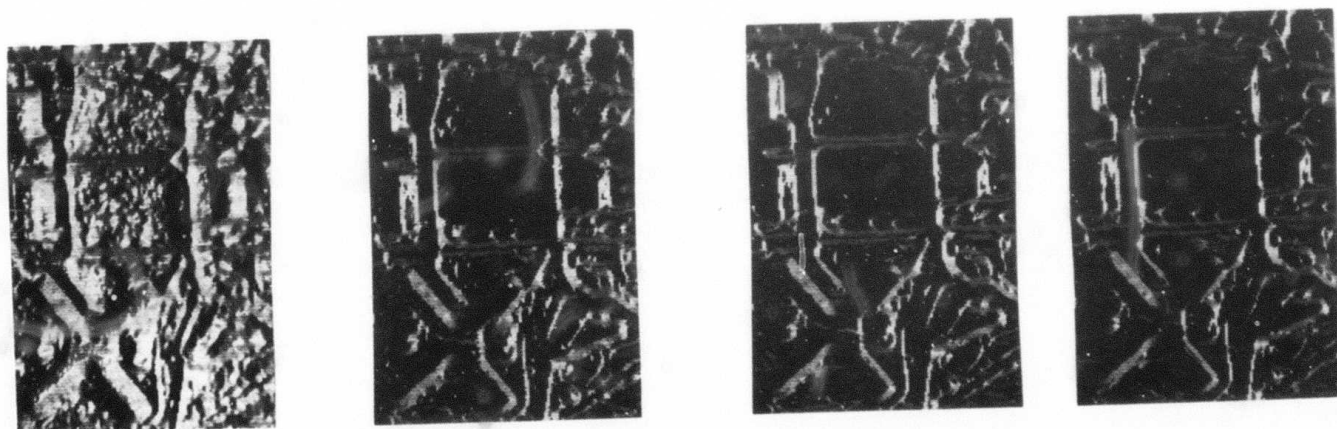
Figure 1. (c) Gradient directions, displayed as gray levels ranging from black to white as the direction (from dark to light) varies from 0° to ±180°; originals and three iterations of enhancement

```
        20  19  18  18  17  18  19  20  20  20  19  18  18  19  19  19  19  19  19  19
        20  20  19  18  19  19  20  20  20  20  19  19  19  19  19  19  18  18  19  20
        20  20  20  20  21  21  20  20  20  20  20  19  19  19  19  19  18  18  19  19
        20  20  21  22  22  21  21  20  20  21  21  20  20  20  20  19  19  19  19  19
        21  21  22  22  23  22  21  21  21  22  22  22  21  20  21  20  20  20  21  20
        24  23  23  24  24  23  23  23  24  24  25  24  23  22  23  23  23  23  24  23
        28  28  28  28  28  28  29  29  30  30  30  30  29  29  29  29  29  30  30  30
        34  34  34  35  35  35  35  36  36  36  36  36  36  36  36  36  36  36  36  36
        38  38  38  38  38  38  38  38  39  39  38  39  38  38  38  38  38  38  38  38
(d1)    36  36  36  36  36  36  36  36  36  35  35  35  35  35  35  35  35  35  34  34
        30  30  31  30  29  29  29  29  29  28  28  28  29  29  29  29  29  29  28  27
        25  26  26  25  25  25  24  25  24  23  23  24  24  25  26  25  26  25  24  24
        24  23  23  23  23  23  23  23  22  22  22  22  23  24  24  23  23  23  22  23
        23  22  22  21  22  21  21  21  21  21  22  22  23  23  22  22  22  22  22  22
        22  21  21  20  20  20  20  20  21  21  21  22  22  21  21  21  21  21  21  21
        21  20  21  20  20  20  20  20  20  21  21  21  21  21  20  20  20  20  21  21
        20  20  20  20  20  20  20  20  20  20  20  20  20  20  20  20  20  20  21  20
        20  20  19  19  19  19  20  20  20  19  19  19  19  19  19  19  20  20  20  19
        20  19  18  18  18  19  19  19  19  19  19  19  19  19  19  20  20  19  19  19
        20  19  19  18  18  19  18  18  18  19  19  19  19  19  19  20  19  19  19  19
```

```
        1   2   2   2   2   2   2   1   1   2   2   2   2   1   1   1   1   0   1   2
        1   1   2   3   4   3   2   1   0   1   1   2   1   1   0   1   1   0   1   0
        1   1   2   4   3   2   1   1   1   1   2   2   1   1   1   1   1   1   1   0
        1   2   2   2   2   2   1   1   2   2   2   2   2   2   2   2   2   2   2   2
        4   3   3   2   2   2   3   3   4   4   4   4   3   3   3   4   4   5   5   5
        7   7   7   6   6   7   8   9   9   9   8   8   9   9   9   9  10  10  10  10
       11  11  11  11  12  12  13  13  13  12  12  12  13  14  14  13  13  13  13  13
       10  10  10  10  10  10  10   9   9   9   9   9   9   9   9   9   9   9   8   8
        3   2   2   2   1   1   1   1   1   0   1   1   1   1   1   1   1   1   1   2
(d2)    7   7   7   8   8   9   9   9  10  10  10  10   9   9   9   9   9   9  10  10
       10  10  10  10  11  11  11  11  11  12  11  11  10  10   9   9   9   9  10  10
        6   7   7   7   6   6   6   6   6   6   6   6   5   5   5   5   6   6   5   4
        2   3   4   3   3   3   3   3   2   1   1   1   1   2   3   3   3   3   2   2
        2   2   2   2   3   3   3   2   1   1   0   1   1   2   2   2   2   1   1   1
        2   1   1   1   1   1   1   1   0   0   0   1   1   2   2   2   2   1   1   1
        1   1   1   1   0   0   0   0   0   1   1   1   2   1   1   1   1   0   0   1
        1   1   1   1   1   0   0   0   0   1   2   2   2   1   1   0   0   0   1   1
        2   1   1   2   1   1   1   1   1   1   1   1   1   1   0   0   0   1   1   1
        2   2   1   1   0   0   1   2   1   0   0   0   0   0   1   1   1   1   1   1
        2   2   1   1   1   1   1   0   0   0   0   0   0   0   0   1   1   1   1   1
```

```
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   7   8   7   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   7   8   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
       10   0   0   0   0   0   0   0  12  12  11  10   0   0   0   0   0   0   0  14
       19  19  17  17  16  19  21  23  23  22  21  22  22  23  23  24  25  25  25  25
       26  26  26  26  26  28  28  30  29  28  27  28  29  30  30  30  30  30  29  29
       22  22  22  22  22  23  22  22  21  20  19  20  20  20  20  20  20  20  19  18
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
(d3)   14  14  15  16  17  18  19  19  20  22  21  20  20  19  19  19  18  19  20  21
       22  22  23  23  23  24  25  24  26  25  26  24  24  21  21  22  21  22  22  22
       17  17  18  18  17  17  17  17  17  16  16  15  15  13  14  15  16  15  14  13
        0   9  10  10   0   9  10  10   8   0   0   0   0   0   8   9   7   0   0   0
        0   0   0   0   5   6   6   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
```

Figure 1.   (d) Gradient magnitudes displayed numerically on a scale of 0-63 for a small subwindow,
indicated by tick marks in (a):   (d1) subwindow gray levels; (d2) original gradient
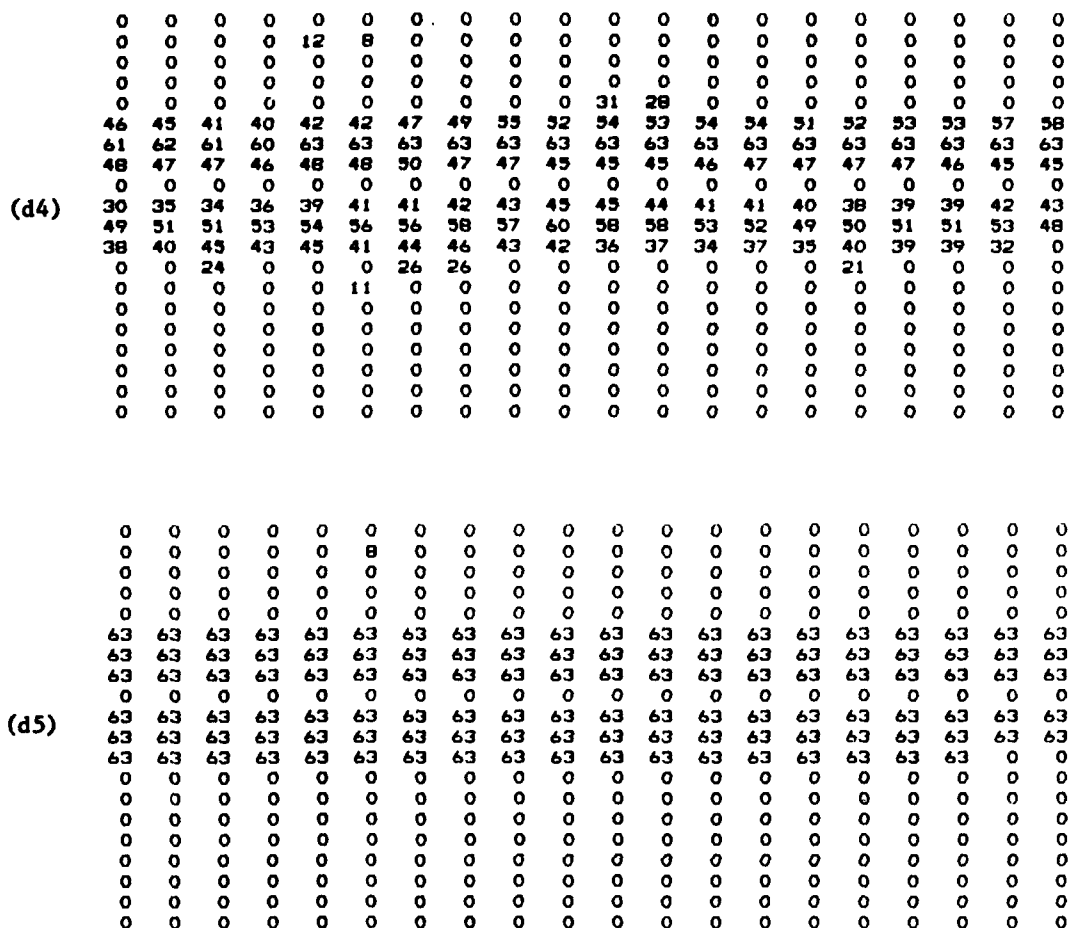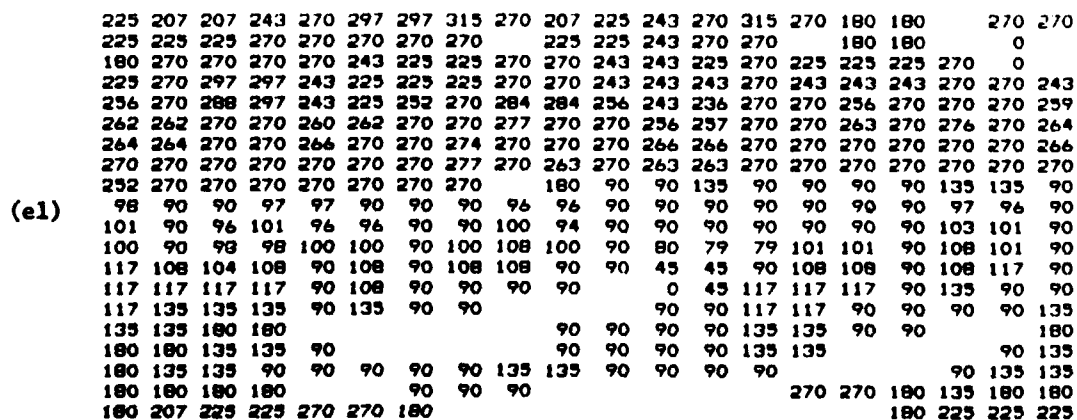magnitudes; (d3-5) results of three iterations

```
                0    0    0    0    0    0    0.   0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0   12    8    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0   31   28    0    0    0    0    0    0    0    0
               46   45   41   40   42   42   47   49   55   52   54   53   54   54   51   52   53   53   57   58
               61   62   61   60   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63
               48   47   47   46   48   48   50   47   47   45   45   45   46   47   47   47   47   46   45   45
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   (d4)        30   35   34   36   39   41   41   42   43   45   45   44   41   41   40   38   39   39   42   43
               49   51   51   53   54   56   56   57   60   58   58   53   52   49   50   51   51   53   48
               38   40   45   43   45   41   44   46   43   42   36   37   34   37   35   40   39   39   32    0
                0    0   24    0    0    0   26   26    0    0    0    0    0    0    0   21    0    0    0    0
                0    0    0    0    0   11    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
```

```
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    8    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
               63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63
               63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63
               63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   (d5)        63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63
               63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63
               63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63   63    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
                0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
```

Figure 1(d), continued

```
              225  207  207  243  270  297  297  315  270  207  225  243  270  315  270  180  180       270  270
              225  225  225  270  270  270  270  270       225  225  243  270  270       180  180       0
              180  270  270  270  270  243  225  225  270  270  243  243  225  270  225  225  225  270   0
              225  270  297  297  243  225  225  225  270  270  243  243  243  270  243  243  243  270  270  243
              256  270  288  297  243  225  252  270  284  284  256  243  236  270  270  256  270  270  270  259
              262  262  270  270  260  262  270  270  277  270  270  256  257  270  270  263  270  276  270  264
              264  264  270  270  266  270  270  274  270  270  270  266  266  270  270  270  270  270  270  266
              270  270  270  270  270  270  270  277  270  263  270  263  263  270  270  270  270  270  270  270
              252  270  270  270  270  270  270  270       180   90   90  135   90   90   90   90  135  135   90
   (e1)        98   90   90   97   97   90   90   90   96   96   90   90   90   90   90   90   90   97   96   90
              101   90   96  101   96   96   90   90  100   94   90   90   90   90   90   90   90  103  101   90
              100   90   98   98  100  100   90  100  108  100   90   80   79   79  101  101   90  108  101   90
              117  108  104  108   90  108   90  108  108   90   90   45   45   90  108  108   90  108  117   90
              117  117  117  117   90  108   90   90   90   90        0   45  117  117  117   90  135   90   90
              117  135  135  135   90  135   90   90                      90   90  117  117   90   90   90   90  135
              135  135  180  180                                90   90   90   90  135  135   90   90            180
              180  180  135  135   90                          90   90   90   90  135  135                 90  135
              180  135  135   90   90   90   90   90  135  135   90   90   90   90                      90  135  135
              180  180  180  180                 90   90   90                            270  270  180  135  180  180
              180  207  225  225  270  270  180                                               180  225  225  225
```

Figure 1. (e) Gradient directions displayed in degrees for the subwindow: original

```
                267 273 276
                273 270

       260                              278 262 252 249                            264
(e2)   264 266 270 269 266 266 271 271 276 271 269 262 262 269 271 270 271 273 270 269
       267 269 270 270 269 271 273 274 270 271 267 269 269 269 270 271 271 271 271 270
       269 270 270 271 271 271 271 274 270 269 270 267 269 270 270 270 270 270 270 271

        98  94  91  93  94  90  90  91  93  93  91  90  90  91  90  90  91  93  93  93
        97  91  93  96  94  93  90  91  94  93  90  89  89  90  90  89  91  97  97  91
        96  91  96  97  98  94  91  96 100  97  90  84  82  82  93  94  93 101 100  91
           105 100  98         100  96 103 103                      101 101 100
                     98  96  96



                280 287

                                        266 262
       264 264 270 270 269 269 271 271 277 274 270 264 263 266 271 271 271 273 274 274
       269 269 270 270 270 273 273 274 271 271 269 269 270 270 271 271 271 271 271 270
(e3)   271 270 270 271 271 271 273 273 271 269 270 270 270 271 270 271 271 271 270 271

        98  91  90  91  91  89  90  90  90  91  90  90  90  90  89  90  90  91  91  91
        94  91  91  94  93  91  90  91  93  91  90  89  89  89  89  89  91  94  94  96
        90  89  94  97  96  89  91  94  98  96  89  86  84  82  87  91  96  98  94
                96                  97 100                              96
                    105



                270

       264 269 271 270 270 271 273 271 274 276 274 264 264 270 271 271 271 271 273 276
(e4)   270 270 270 271 271 273 273 274 273 273 270 270 270 270 271 271 271 273 273 271
       271 270 270 271 271 271 273 273 271 270 270 270 270 271 271 271 271 271 271 271

        94  90  90  90  90  89  90  90  90  90  90  90  90  90  89  89  90  90  91  91
        93  90  90  93  91  91  89  91  91  91  90  89  87  87  87  89  91  93  97  96
        91  87  91  97  93  86  87  96  97  91  89  87  86  86  84  90  96  93
```

Figure 1.   (e) Gradient directions displayed in degrees for the subwindow: three iterations

EEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEE
EEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEE
EEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEE

vvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvv
vvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvv
vvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvv

Figure 2.   (a) Edge components extracted from the subwindow in Figure 1

Figure 2.   (b) Line segments fitted to the edge components in the window
                 of Figure 1

Figure 3.   Results of collinear linking (heavy lines) for the window of Figure 1

(a)

(b)

Figure 4.   (a) Results of antiparallel linking (heavy lines, joined by dashed lines) for the
window of Figure 3.   (b) The antiparallel pairs only

(a)

(b)

(c)

Figure 1'. Analogous to Figure 1 for a second window showing part of a new suburban area

```
(d1)
28 27 26 28 29 29 26 21 20 20 22 25 28 29 29 28 26 23 22 22 24 24 25 26
27 25 23 23 25 27 26 24 23 24 25 27 28 29 28 27 25 23 23 24 23 23 24 26
24 22 21 23 25 27 30 31 30 30 29 28 27 27 26 24 23 22 23 25 25 24 24 25
21 21 23 26 28 31 34 35 35 35 33 29 27 26 26 24 23 22 23 25 26 25 25 26
19 21 26 31 34 35 36 37 38 37 35 31 29 28 28 27 25 24 24 26 27 27 25 26
19 21 27 31 35 37 38 38 39 39 38 34 32 31 32 31 29 28 27 28 29 28 26 27
19 20 22 27 33 38 39 40 40 41 40 37 35 35 36 36 34 31 30 30 30 29 26 24
19 18 19 23 32 39 41 41 41 41 41 40 40 39 40 39 38 36 33 31 30 28 23 18
18 19 19 23 33 40 42 42 42 42 42 42 42 42 42 41 41 40 37 33 30 26 19 14
17 18 19 23 33 40 42 43 43 43 43 43 43 43 42 42 41 39 35 30 25 18 13
14 16 18 24 34 41 43 43 43 43 43 43 43 43 43 43 42 42 40 36 29 25 18 13
12 13 18 25 35 41 43 43 43 44 43 43 44 44 43 43 43 42 40 36 30 25 18 13
12 13 18 25 36 42 43 43 43 43 44 44 44 44 44 43 43 43 42 41 36 31 26 19 14
15 15 18 25 35 42 43 43 43 44 43 43 43 43 43 43 43 42 40 37 31 26 20 17
19 20 20 25 35 41 43 43 43 43 43 43 43 43 43 42 42 42 40 36 30 26 22 20
24 24 23 27 35 39 42 42 42 42 42 42 43 42 42 42 42 40 39 34 29 26 24 22
27 27 27 28 31 35 39 39 39 38 39 39 40 41 40 40 39 38 36 33 29 26 25 24
29 29 28 29 31 34 36 35 34 34 34 36 36 37 37 36 34 33 32 31 29 28 25 24
30 30 31 34 36 37 36 33 31 31 32 33 33 33 34 35 35 32 30 30 30 29 28 25 23
34 34 36 39 40 40 37 34 32 33 33 33 32 34 36 36 33 31 30 30 30 28 25 23
39 40 40 41 42 42 40 38 36 36 36 36 36 37 38 38 36 34 33 33 33 32 29 28
42 42 43 43 43 43 42 41 40 40 40 40 40 40 40 40 39 39 38 38 38 37 36 35
40 42 43 43 44 43 43 42 42 42 42 42 42 42 41 41 41 41 41 41 41 40 39 38
38 40 42 43 43 43 43 43 43 43 42 43 43 43 42 42 42 42 42 42 42 42 41 39
```

```
(d2)
1  2  4  6  5  4  7  6  4  5  6  6  4  1  1  4  6  4  3  3  1  1  2  1
3  5  5  4  4  1  5  9 10 10  7  3  1  2  3  4  4  2  1  2  2  0  2  2
5  3  1  4  5  6  8 11 12 11  8  3  1  2  3  3  3  1  2  2  3  2  1  2
3  2  7  9  9  8  7  7  8  7  7  6  3  2  2  4  4  2  2  2  3  2  2  2
1  6  8  9  8  6  5  4  4  5  7  7  5  6  6  7  7  6  5  4  3  3  2  2
2  6  9  8  6  3  3  3  3  4  7  8  7  7  8  9  9  8  6  5  3  3  2  2
2  4  9 11 11  6  3  3  3  3  5  7  8  8  9  9  8  6  4  2  5  6  8
2  1  5 12 15  8  3  3  2  2  3  5  7  7  6  6  7  9  8  5  4  8 11 11
3  1  4 13 16  9  2  2  2  2  3  3  4  4  4  4  4  6  9  8  7 11 12  8
4  2  5 14 17  9  2  1  1  1  1  1  1  1  2  2  2  3  7  9 10 12 12  7
4  5  8 15 16  8  2  1  1  1  0  1  1  1  1  1  1  3  6 11 11 12 12  7
1  5 11 17 16  8  1  0  1  1  1  1  1  1  1  1  1  3  6 11 11 12 12  7
3  5 11 17 16  7  1  0  0  1  1  0  0  1  1  0  1  3  6 10 11 12 12  8
7  6  9 16 16  7  1  0  0  0  1  1  1  1  1  1  1  3  6 10 11 11  9  7
9  8  7 14 15  7  2  1  1  1  1  1  0  0  1  1  2  3  6 10 10  8  8  7
8  8  6 10 11  8  5  4  4  4  4  3  3  2  2  3  3  4  6 10  9  6  5  5
5  5  5  5  7  8  6  7  7  8  7  6  6  5  5  6  7  7  7  7  7  5  3  2
3  4  5  6  7  4  2  5  7  7  6  6  6  5  5  7  7  5  5  4  4  4  2
6  6  8 10  9  6  4  5  2  1  1  2  3  2  1  3  5  3  1  2  3  5  5  2
10 10  9  8  6  5  7  7  5  5  4  4  3  4  3  5  6  5  4  4  4  6  7  5
8  8  7  5  4  4  6  8  8  8  7  8  8  6  5  5  7  8  8  8  9 10 11 12
2  2  3  3  2  2  4  5  6  6  6  6  5  4  4  3  7  8  8  8  9 10 10
4  2  1  0  0  1  1  2  3  3  3  3  3  3  3  3  4  4  4  5  5  5  6
3  3  2  1  0  1  1  1  1  1  1  1  2  2  2  2  2  2  2  2  2  2  4  6
```

```
(d3)
0  0  0 11  8  0  0 13 10 13 16 15 10  0  0  9 11  8  6  0  0  0  0  0
6 10  8  7  0  0  0 19 24 23 16 10  0  0  7  9  8  0  0  0  0  0  0  0
9  8  0  0  0 12 18 24 28 25 16 10  0  0  4  6  5  0  0  0  5  0  0  0
6  0  0 19 19 20 17 19 20 19 19 14  9  0  0  0  0  0  0  6  0  0  0
0  0 18  0 14 15 12 12 12 15 18 18 15 12 15 15 16 14 11  9  8  5  0  0
0  0 20 20 16 12  8  8  8 11 16 19 17 18 20 21 20 18 15 10  8  8  0  0
0  0 20 26 23 15  9  8  7  8 13 17 18 20 20 20 22 21 15  9  0 11  0 20
0  0 15 28 32 21 10  6  0  0  9 12 16 17 16 15 18 21 19 14  0 18 24 25
6  0  0 31 37 24  0  0  0  0  5  8  9 11 10  9 11 17 20 19  0 27 28 22
10  0  0 33 36 23  0  0  0  0  0  0  0  0  0  0  0 11 18 22 27 30 29 19
9 13  0 37 38 23  0  0  0  0  0  0  0  0  0  0  0  8 17 26 30 31 28 19
0 12  0 39 36 21  0  0  0  0  0  0  0  0  0  0  0  8 17 25 30 32 29 19
7 13 29 41 38 21  0  0  0  0  0  0  0  0  0  0  0  8 17 25 29 30 27 18
16 17 22 38 37  0  0  0  0  0  0  0  0  0  0  0  0  8 16 24 28 27 24 20
21 20  0 31 33 20  0  0  0  0  0  0  0  0  0  0  9 16 24 26 22 18 16
20 18 17 22 26 20 13  9 11  0  0  0  7  0  0  7 10 13 17 22 21 16  0 10
13 14  0 15 16  0 12 14 17 17 16 14 13 12 11 12 14 16 18 17 17 12  9 10
10  0 14 14  0  0  0  0 14 14 14 14 13 12 10 11 12 14 13  0 11 12  9  0
17 16 19 21 18  0  0  0  0  0  0  0  0  0  0  8  0  0  8 11  0  0
23 22 22 19 16 13 16 16 12 10  0  8  8  6  7  8 11 10 10 10 12  0 18 14
18 18 16 13 10 10 15 17 18 17 17 16 16 14 10 11 16 17 18 19 19 21 24 24
0  0  0  6  0  0  0 12 15 15 15 15 15 14 10 10 13 16 18 19 19 21 23 21
5  6  0  0  0  0  0  0  0  9  0  0  9  8  7  7  9 10 12 13 12 13 15 17
5  6  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 10 14
```

Figure 1'.   (d1-3)

```
 0   0   0   0  10   0   0  28  24  29  40  37  23   0   0  19  22   0  10   0   0   0   0   0
 0  13  18   0   0   0   0   0  55  54  40  21   0   0   0  15  17   0   0   0   0   0   0   0
24  15   0   0   0   0  41  52  63  59  40  20   0   0   0   0   0   0   0   0   0   0   0   0
18   0   0   0  29  38  44  47  51  48  46  38  22   0   0   0   0   0   0   0   0   0   0   0
 0   0   0   0  33  36  35  32  33  39  47  45  35  34  31  34  31  28  23  20   0   0   0   0
 0   0  36  30  32  30  26  20  22  29  43  46  42  47  49  52  50  44  34  24  18   0   0   0
 0   0  34  55  58  40  26  18  16  19  32  41  45  49  49  51  51  47  38   0   0  21   0  43
 0   0   0  63  63  51  24   0   0   0  18  29  39  42  40  39  45  52  49   0   0  30  50  62
 0   0   0  63  63  58   0   0   0   0   0   0  23  24  23  22  30  44  51  47   0  62  62  55
18   0   0  63  63  57   0   0   0   0   0   0   0   0   0   0   0  29  47  54   0  63  63  48
 0   0   0  63  63  53   0   0   0   0   0   0   0   0   0   0   0  20  43  63  63  63  63  51
 0  31   0  63  63  54   0   0   0   0   0   0   0   0   0   0   0  21  45  63  63  63  63  46
19  21   0  63  63   0   0   0   0   0   0   0   0   0   0   0   0  21  43  62  63  63  63  48
42  44  51  63  63   0   0   0   0   0   0   0   0   0   0   0   0  21  43  61  63  63  55  45
52  35   0  63  63  33   0   0   0   0   0   0   0   0   0   0   0  24  43  60  63  50  38  41
47  34  35  46  52  39  31  22   0   0   0   0   0   0   0   0  21  32  42  55  55  42   0   0
33   0   0  32   0   0   0  33  29  36  31  29  25  25  25  28  32  34  41  35  37  32   0   0
21   0  23  29   0   0   0   0   0  29  28  27  26  24  24  21  25  32  27   0  26  29  20   0
38  43  42  40   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  10  25  28   0
54  53  49  43  33  34  23  21  22   0   0   0  18  18   0  17  21  26  24  25   0   0  34  34
41  39  37  28  22  20  32  41  39  38  37  35  35  30  27  25  29  37  41  43  45  49  45  56
 0   0   0   0   0   0   0   0  34  32  34  35  33  31  27  25  30  40  44  45  48  51  54  50
 0   0   0   0   0   0   0   0   0   0   0   0   0  19  17  17  19  24  27  28  29  34  38  37
 0   4   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  27  37
```

(d4)

```
 0   0   0   0   0   0   0   0  54  63  63  63  50   0   0  37  48   0   0   0   0   0   0   0
 0   0  17   0   0   0   0   0   0  63  63  51   0   0   0   0   0   0   0   0   0   0   0   0
44  35   0   0   0   0   0  63  63  63  63  49   0   0   0   0   0   0   0   0   0   0   0   0
41   0   0   0   0  63  63  63  63  63  63  63  54   0   0   0   0   0   0   0   0   0   0   0
 0   0   0   0   0  63  63  63  63  63  63  63  63  63  63  63  63  63  61  51   0   0   0   0
 0   0   0   0  63  63  63  58  56  63  63  63  63  63  63  63  63  63  63  46  31   0   0   0
 0   0   0  63  63  63  63  46  29  47  63  63  63  63  63  63  63  63   0   0   0  29   0   0
 0   0   0  63  63  63  53   0   0   0   0  63  63  63  63  63  63  63   0   0   0  63  63  63
 0   0   0  63  63  63   0   0   0   0   0   0  57  55  53  63  63  63   0   0  63  63  63
 0   0   0  63  63  63   0   0   0   0   0   0   0   0   0   0  63  63  63   0  63  63  63
 0   0   0  63  63  63   0   0   0   0   0   0   0   0   0   0   0  52  63  63  63  63  63  63
 0   0   0  63  63   0   0   0   0   0   0   0   0   0   0   0   0  53  63  63  63  63  63  63
38  59   0  63  63   0   0   0   0   0   0   0   0   0   0   0   0  55  63  63  63  63  63  63
63  63  63  63  63   0   0   0   0   0   0   0   0   0   0   0   0  54  63  63  63  63  63  63
63  63   0  63  63  63   0   0   0   0   0   0   0   0   0   0   0  60  63  63  63  63   0   0
63  63  54  63   0   0  63   0   0   0   0   0   0   0   0   0  46  63  63  63  63  63   0   0
48   0   0  63   0   0   0   0  63  50  63  55  52  48  47  62  63  63  63  63  63  63   0   0
 0   0  59   0   0   0   0   0   0   0  60  54  51  52  50  52  56  58   0   0  54  63   0   0
63  63  63   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  56  63   0
63  63  63  63  63  59  60  49   0   0   0   0   0   0   0   0   0   0  39  54   0   0  63  63
63  63  63  62  49  49  51  63  63  63  63  63  63  63  62  52  63  63  63  63  63  63  63  63
 0   0   0   0   0   0   0   0   0  63  63  63  63  63  63  63  63  63  63  63  63  63  63   0
 0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  42  42  48  56  63  63  63  63  63
 0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  57  63
```

(d5)

Figure 1'. (d4-5)

```
315 135  90  72  79 135 180 198 243 281 308 322 315 315 180 180 190 207 252 288 315  45  45  45
108 135 101  63  45  45 233 252 264 276 285 304   0  90 124 135 153 153 315 297 243   0   0
113 124   0 315 315 308 284 264 266 264 256 236 180 117 146 146 146 135   0 297 252 243 315 333
108   0 315 302 294 297 293 278 263 255 231 218 214 243 225 225 225 243 315 315 270 236 270 333
 90 350 333 319 300 297 293 284 270 239 219 219 239 260 260 248 248 260 281 297 270 236 243 333
 27  10  18  21   0 326 288 288 270 243 225 225 248 270 270 252 246 249 270 281 270 214 207  90
 27  45  41  31  15 350 288 270 270 252 225 231 256 270 270 252 246 243 252 256 225 180 153 117
 27  45  23  10   0 346 304 270 270 270 236 248 262 270 270 252 248 238 235 225 180 159 149 128
 18   0   0 356 356 353 315 270 270 270 252 252 256 270 256 243 243 232 221 210 188 170 162 145
 45  45  11 356 357 353 333 270 270 270 270 243 243 243 214 203 193 180 176 176 172
 63  45   7 353 353 346 333 270 270 225     270 270 225 225 225 225 198 190 186 186 180 180 180
 45  23   0 357 353 353 315     270 225 270 270 270 225 225 225 225 198 190 186 190 190 184 188
270 338 354   0 356 352 315         270 225             180 180     180 190 186 186 190 194 210
270 297 342 356   0   0   0         135  90  90 180 180 180 180 180 180 180 180 186 198 231
270 277 309 347   0   8  27  90  90  90 135  90         180 135 153 162 170 169 174 187 210 236
277 263 297 343  10  35  53  90 104  90  90  90  72  90 117 108 124 135 153 169 173 190 217 233
281 259 281 338  15  35  63  98  98  90  82  80  90 101 108 113 113 129 158 180 203 214 225
304 256 281 297 315 333  90 121 105  90  80  80  80  80 101 121 124 113 121 143 180 194 194 180
280 270 291 293 288 270 207 180 153  90  45  90  72  45  90 180 169 162 135 180 198 191 180 180
270 276 288 291 280 248 225 225 248 270 270 256 288 315 288 233 218 233 256 256 243 218 225 248
270 277 285 293 270 243 232 240 256 270 270 263 270 288 281 248 236 249 263 270 257 248 250 256
270 297 288 270 270 225 225 239 260 270 270 270 270 270 270 256 248 255 263 270 263 252 253 253
 63  49  49         225 229 243 252 252 270 270 270 252 252 252 252 256 256 270 259 248 239 218
 34  34  27  45     180 225 225 270 225 270 270 270 243 243 270 270 270 270 243 243 243 225 198
```

(e1)

Figure 1'. (e1)

```
(e2)
                80   87              211 239 285 308 316 316        174 186 205 248
      114 125 107  80                  243 266 276 294 304        125 139 169
      121 122                294 284 270 269 266 249 240        160 166 172              257
      124            307 293 301 290 280 264 245 231 224 226                          235
                346      293 288 287 281 267 236 222 222 238 257 260 256 255 263 276 287 276 256
                 13  11 354 329 304 283 267 240 226 228 253 267 266 259 256 257 267 274 264 212
                 38  31   6 343 305 277 273 256 228 232 259 270 270 259 242 239 243 246        183          122
                 27  11   0 350 307 276          236 242 263 267 267 259 242 236 232 224        170 145 129
       25               0 359 354              267 267 266 266 259 255 250 231 222 214        172 165 143
       41             356 356 354                                        218 208 203 181 177 176 170
       55  42         354 356 352                                        198 191 187 184 181 181 181
           25         357 356 353                                        193 187 186 187 187 186 188
      284 328 356   0 359 356                                            183 186 184 186 186 190 215
      276 305 333 359   0                                                177 179 180 180 184 195 226
      273 281         353   3  13                                        163 172 173 176 186 214 235
      274 270 304 346   8  34  55  82  96              79        120 128 135 146 170 177 187          245
      276 267         336   3      68  87  91  91  86  82  83  89  98 103 104 118 131 153 181 197 201
      304          277 288             101  90  82  79  83  86 100 108 110 110 122        187 197 193
      277 280 285 287 283                                                173 165        204 201 193
      271 278 285 298 276 252 229 226 252 267     263 285 295 278 246 207 245 255 257 248      225 252
      271 274 278 283 269 252 233 238 259 267 269 269 269 273 280 276 257 238 245 263 267 260 256 257 260
                269             238 260 267 269 271 269 267 266 260 257 260 266 267 267 264 260 259 257
       75                          262           266 257 257 259 263 264 263 267 262 255 233 221
       21  38                                                                            222 211
```

```
(e3)
                103              203 250 295 308 315 315        170 183      235
       110 110                   267 277 295 304                159 174
      124 128              283 277 270 266 248 242
      127            278 291 287 280 266 243 231 226 228
                312 302 287 280 264 235 225 225 238 256 264 263 263 269 276 280
                 17  23 350 328 308 295 266 238 226 229 253 264 267 262 260 266 276 262
                 18  18   1 343 311 290 276 252 231 233 259 269 270 262 242 239 240        191      128
                 13   1 350 309          232 255 264 269 267 260 240 233 231        188 139 131
                  4 359 353              269 267 263 260 243 229 224 214        170 155 143
       31           359 357 357                                          217 212 207        173 176 169
                  359 356 356                                            195 194 187 184 181 181 181
           23       354 357 357                                          190 186 184 186 186 186 191
      288 346       356 356                                              183 184 183 184 184 190 215
      278 307 328     3   0                                              179 179 180 180 184 194 226
      274 300         354   8  30                                        153 172 174 177 180 217 233
      273 269 300 352   0  38  58  80                                    121 134 145 170 177 198
      280              338          80  77  86  87  89  86  86 100 103 104 111 132 146 188 195
      309          300 308              89  84  83  84  87  93 104 105 108 129        190 197 190
      274 281 287 277                                                    180 215 204
      271 277 283 285 267 255 250 250 257         280 285     245 201 233 255 260          224 256
      271 274 274 276 274 259 238 238 257 263 269 274 271 277 274 257 252 256 264 266 262 259 267 263
                        259 267 273 267 266 267 266 263 262 263 266 267 266 263 260 260
                        262 262 266 270 270 269 269 266 257 235 225
           53                                                                            218 215
```

```
(e4)
                      242 297 309 314 314        176 184
       114                278 297 304
      122 124            280 277 267 246 242
      129            295 288 280 269 242 232 229 231
                307 288 280 263 235 226 226 239 256 266 266 267 271 277
                349 328 311 298 264 238 228 231 255 264 267 264 263 262 271 270 249
                 21   0 343 315 293 271 252 231 235 260 269 270 263 242 238        195
                  8   1 352 318              246 263 269 269 260 239 233 225        188 138 131
                  0   0 354                  269 266 262 240 229 224        176 149 142
                  0 359 359                                      217 215 208        170 176 166
                  0 357 359                                      195 194 194 205 176 181 181
                359 353                                          188 184 184 184 184 186 193
      300 349       352 353                                      183 183 183 184 184 188 215
      281 288 308   0   3                                        179 179 180 180 183 205 224
      277 305         354  10  35                                152 172 176 179 177
      277 269 297 350          62                                124 131 146 158 179 187
      298              339          75  80  89  89  87  89  94 101 104 110 134 145 191 195
                309                    84  84  86  89  93  98 100 104        190 205
      271 281 287                                                217 210
      271 276 278 274 264 262 255 255                            248 260          226 259
      271 273 273 271 270 262 255 252 256 263 269 276 276 270 271 262 257 260 264 263 262 264 271 273
                        267 270 270 266 264 267 266 266 267 267 269 267 264 263
                        264 269 273 273 270 270 269 257 235 224
                                                                                224 219
```

Figure 1'. (e2-4)

(a)

(b)

Figure 2'. Analogous to Figure 2 for the second (sub)window
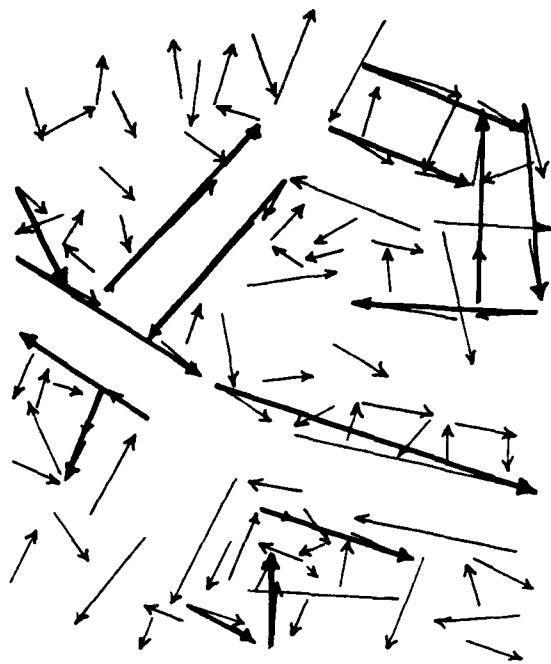
Figure 3'. Analogous to Figure 3 for the window of Figure 1'.
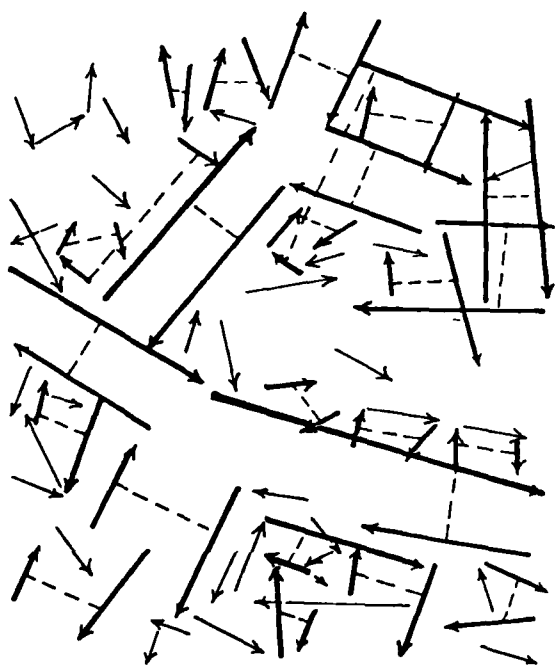
(a)
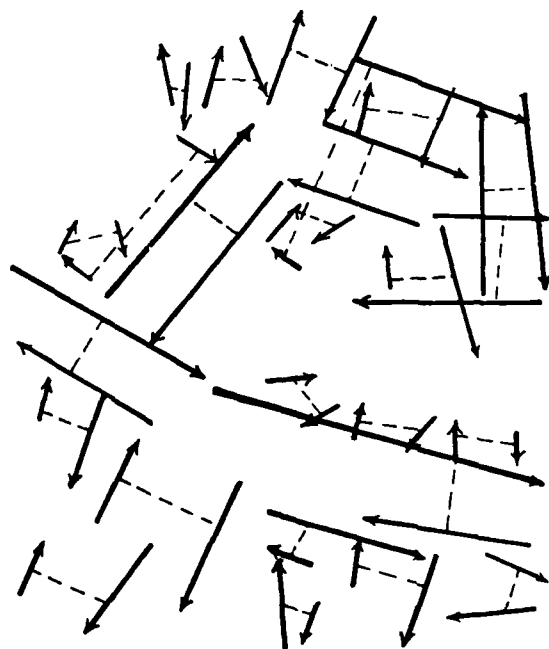
(b)

Figure 4'. Analogous to Figure 4 for the window of Figure 3'.
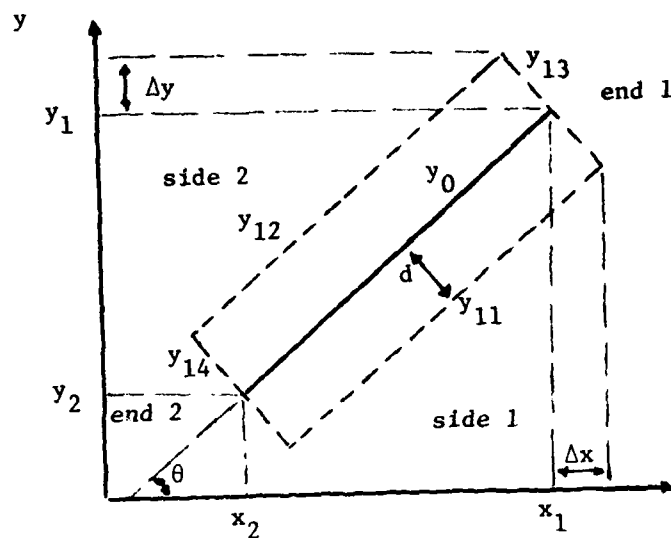
(a)

$0 \leq \theta < 90$
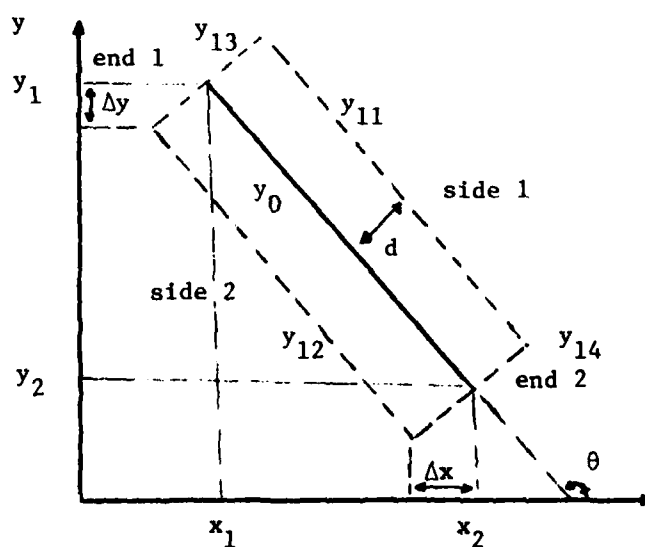
(b)

$90 < \theta < 180$

Figure 5. End and side conventions.
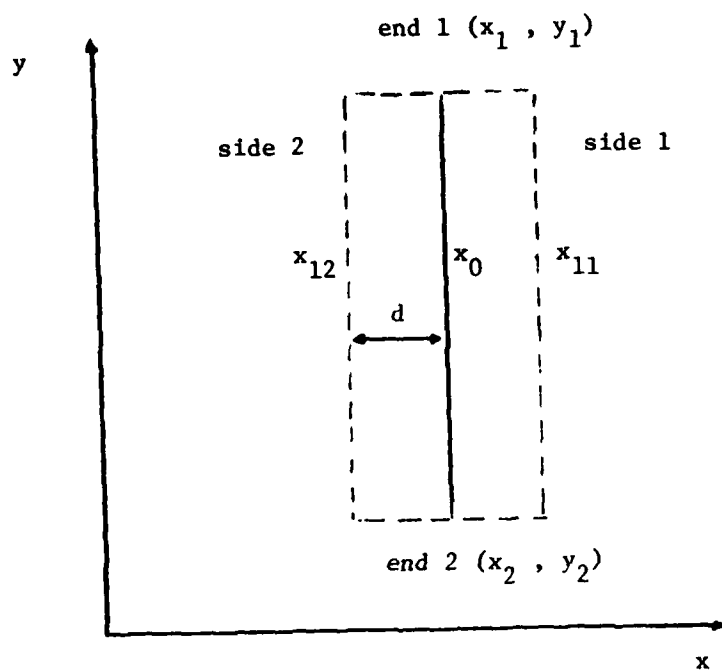
Figure 6. θ=90°
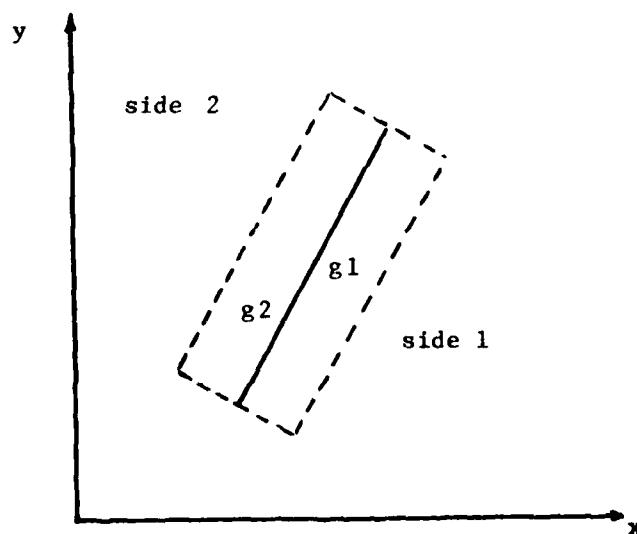
Figure 7. A line segment and strips along each side of it
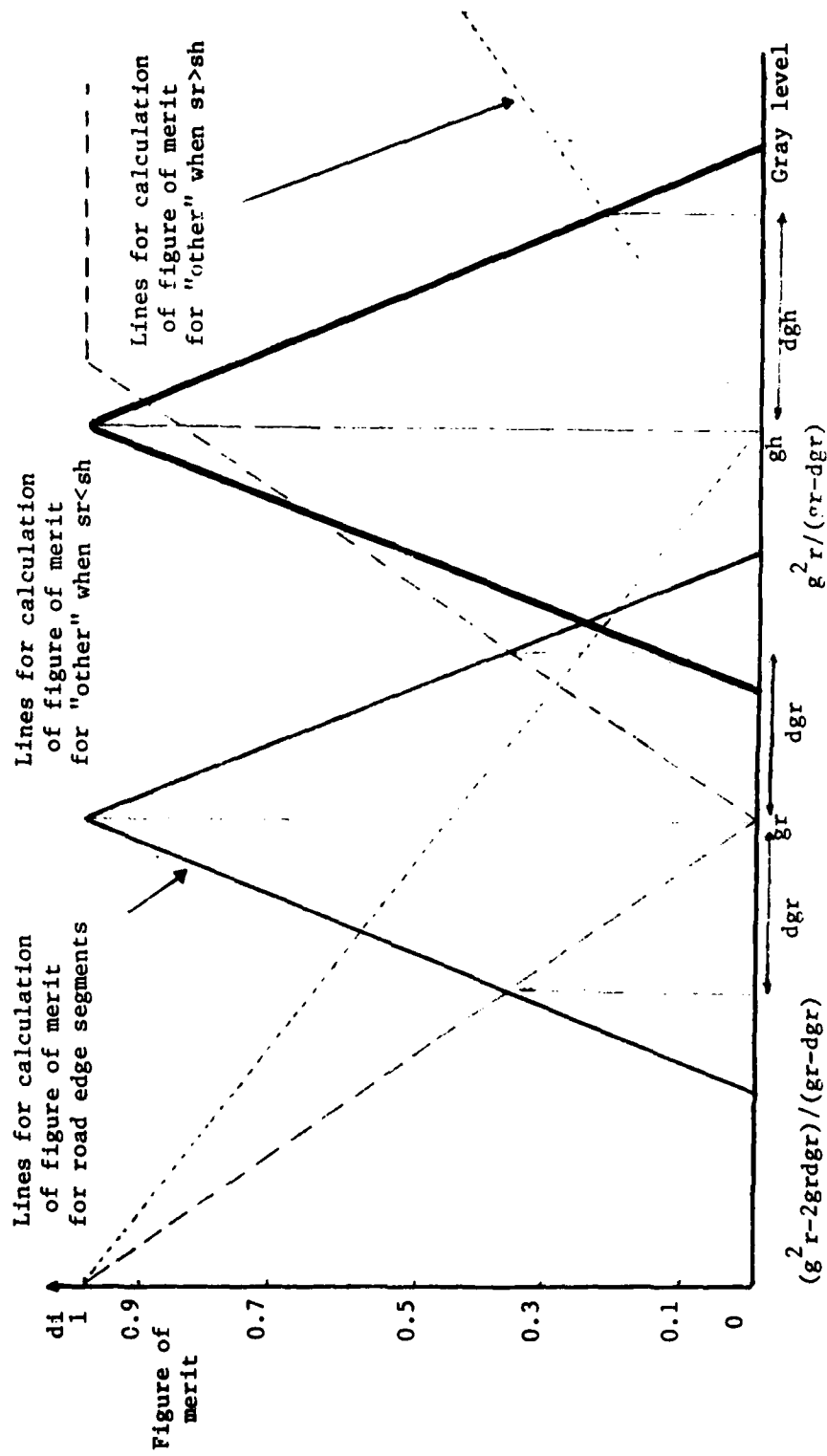
Figure 8. Linear functions for calculation of figures of merit

$\theta = \pi + |\theta_A - \theta_B|$ if $\theta_B \geq \theta_A$

$\theta = \pi - |\theta_A - \theta_B|$ if $\theta_B \leq \theta_A$

(a)

$\theta = |\theta_A - \theta_B|$

(b)

$\theta = 2\pi - |\theta_A - \theta_B|$

(c)
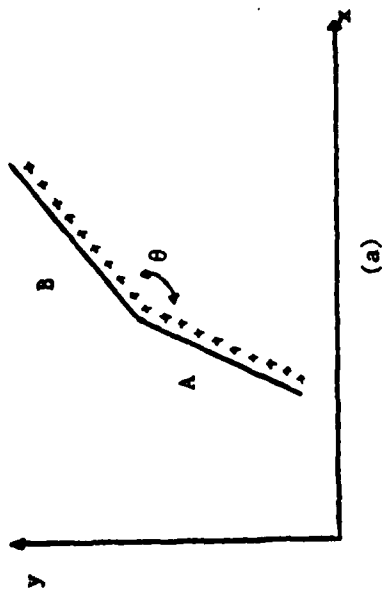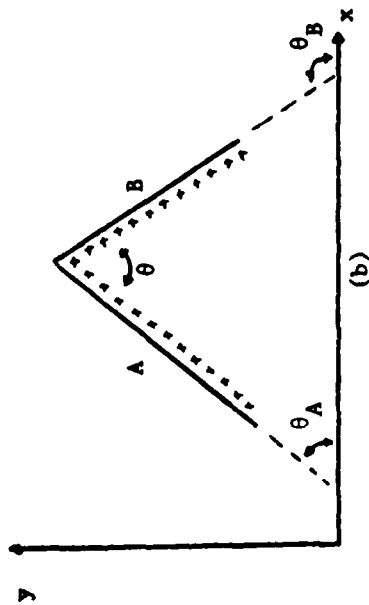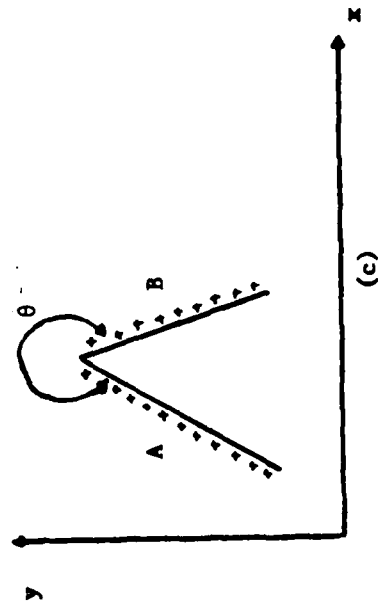
Figure 9. The angle between two lines in different orientations
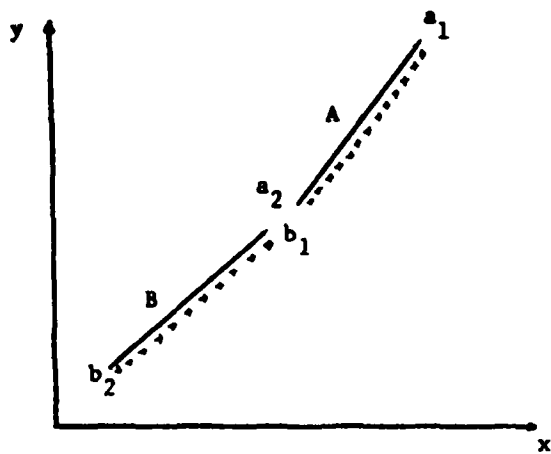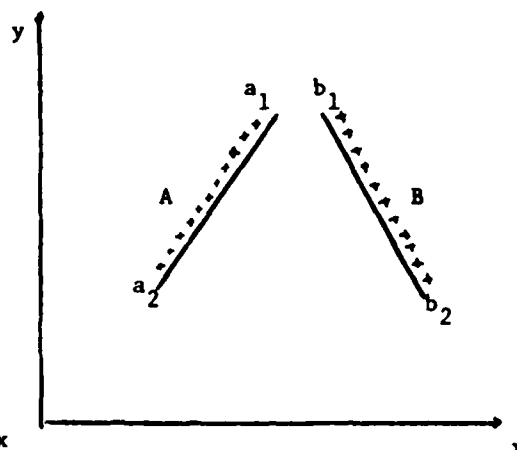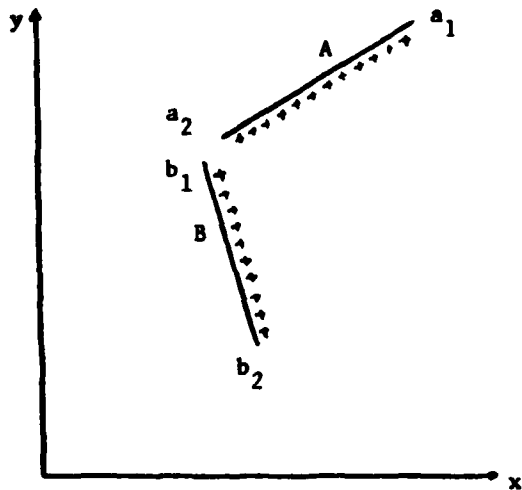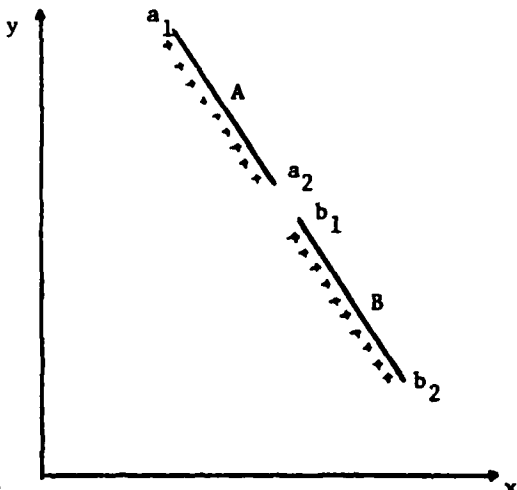
Figure 10. Examples of geometrically compatible candidate pairs.
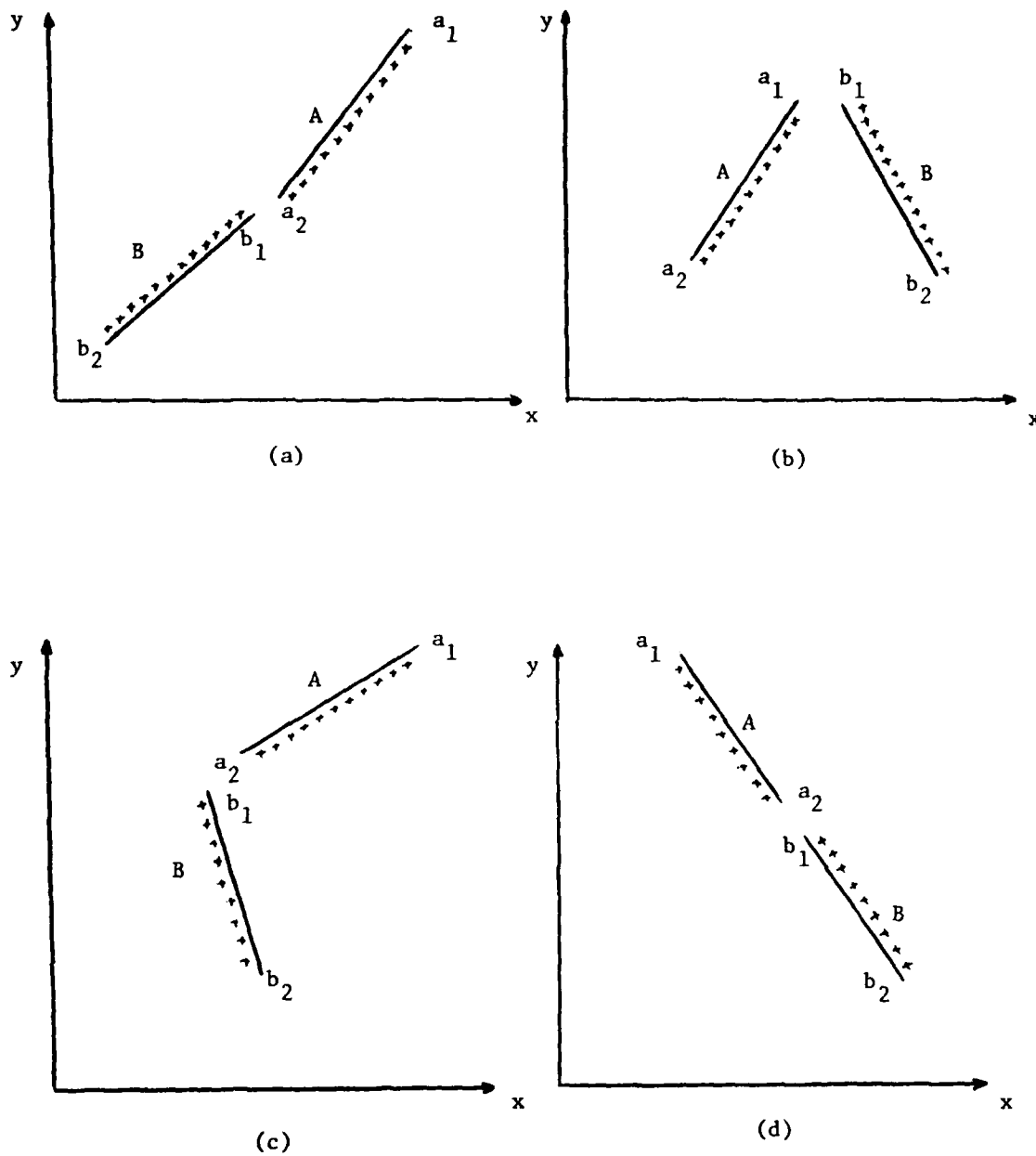
Figure 11. Examples of geometrically incompatible candidate pairs.
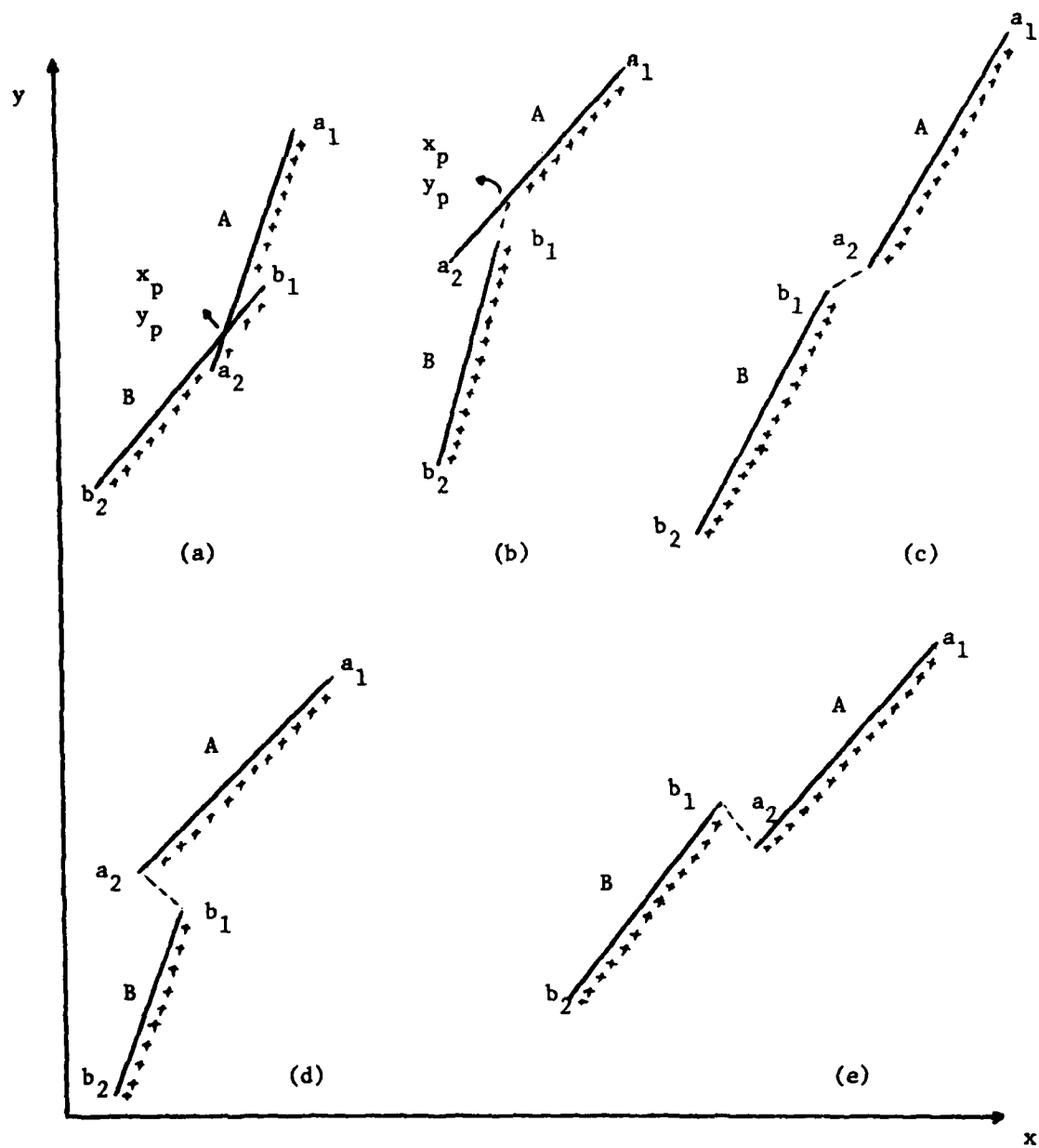
Figure 12. Configurations in Case 1
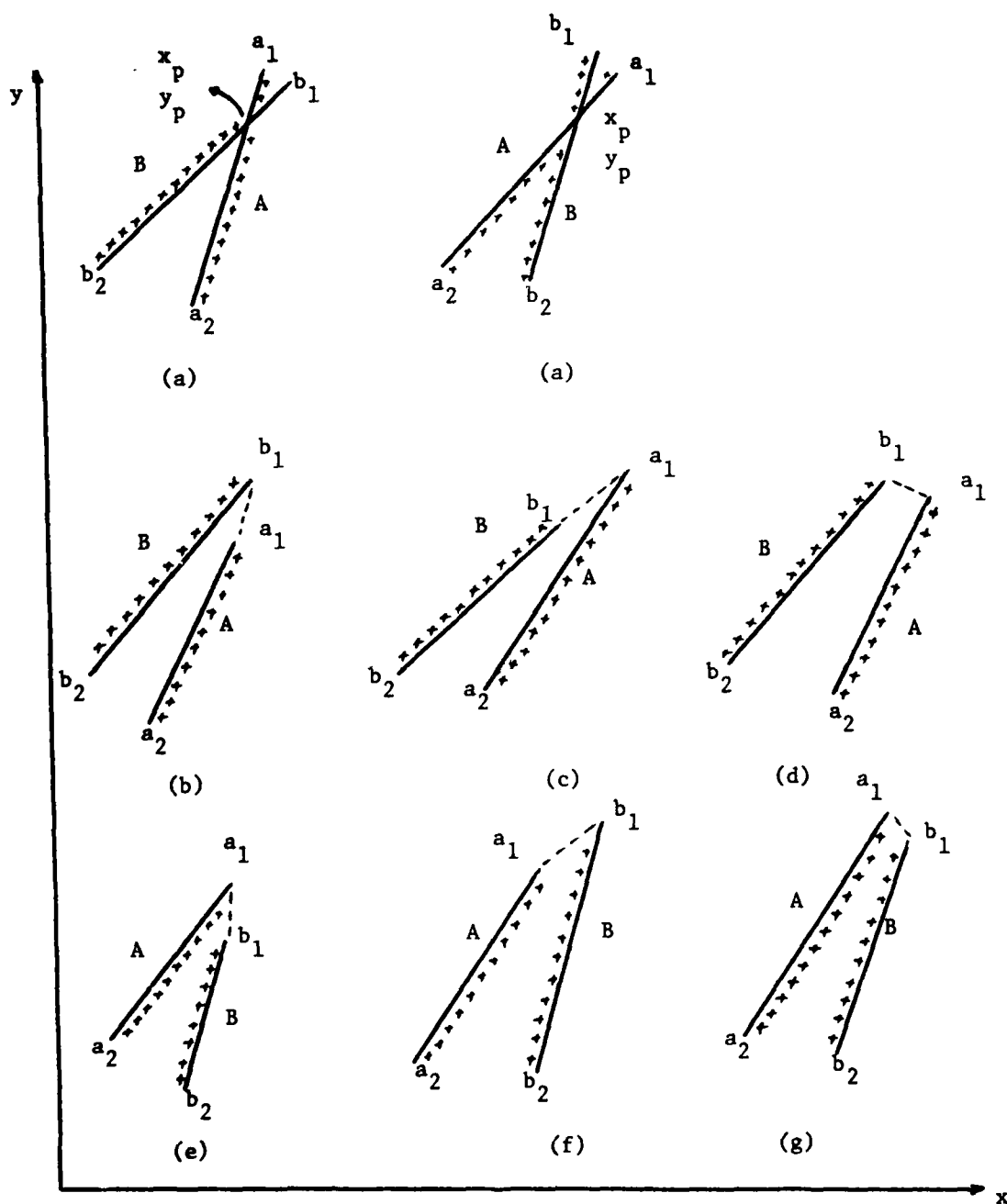
Figure 13. Configurations in Case 2
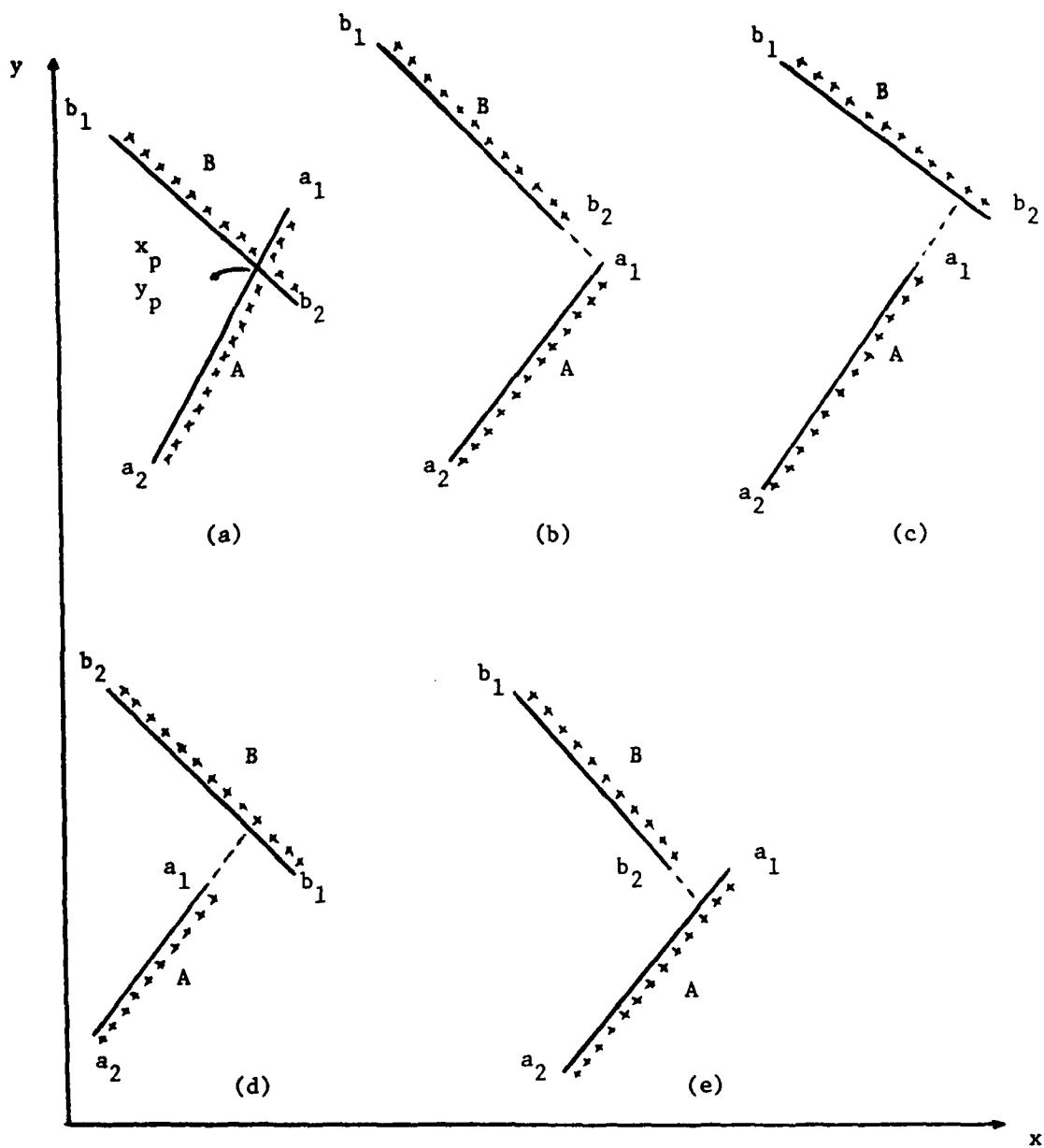
Figure 14. Configurations in Case 3: end 1 of line A
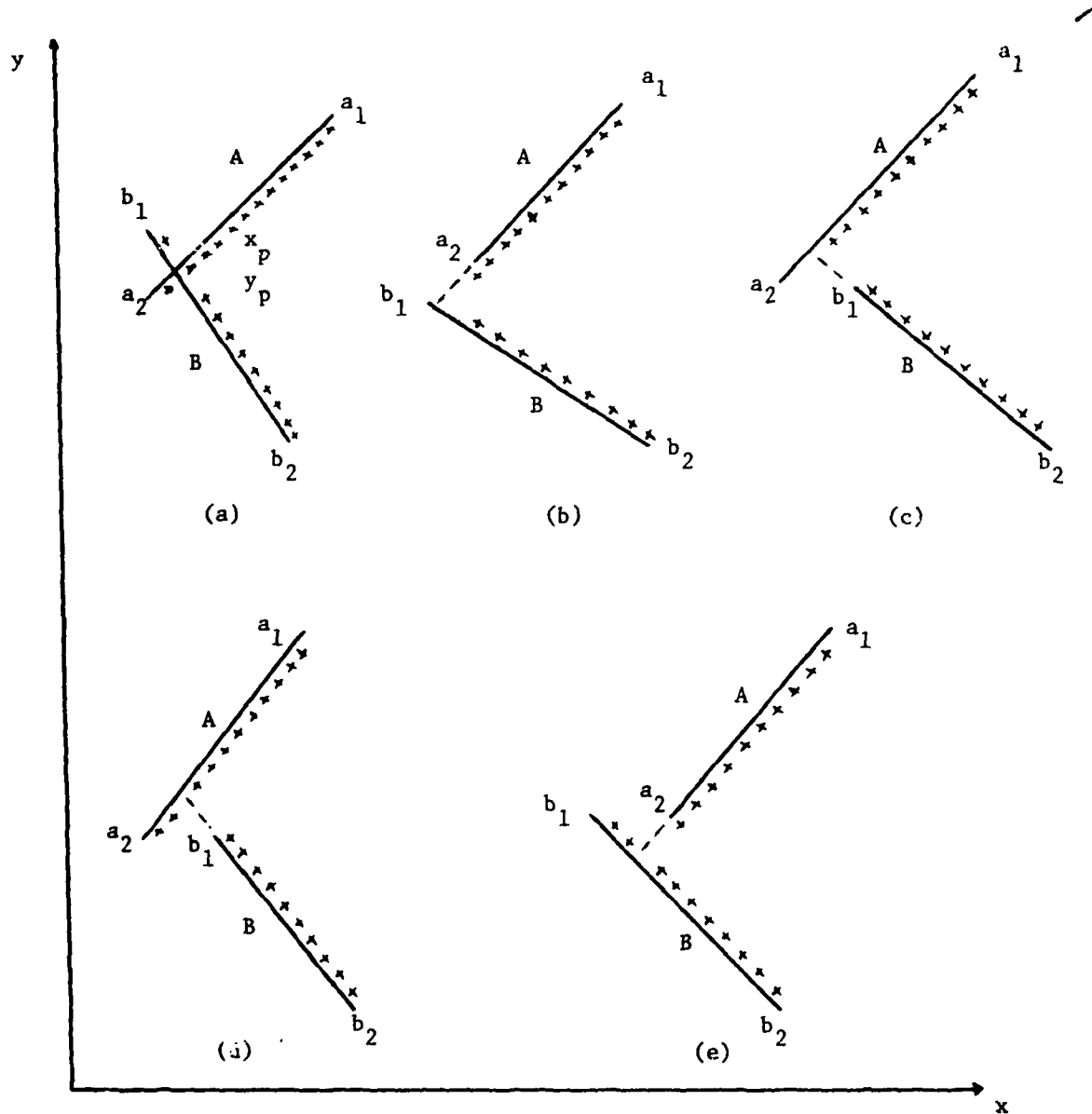
Figure 15. Configurations in Case 3: end 2 of line A
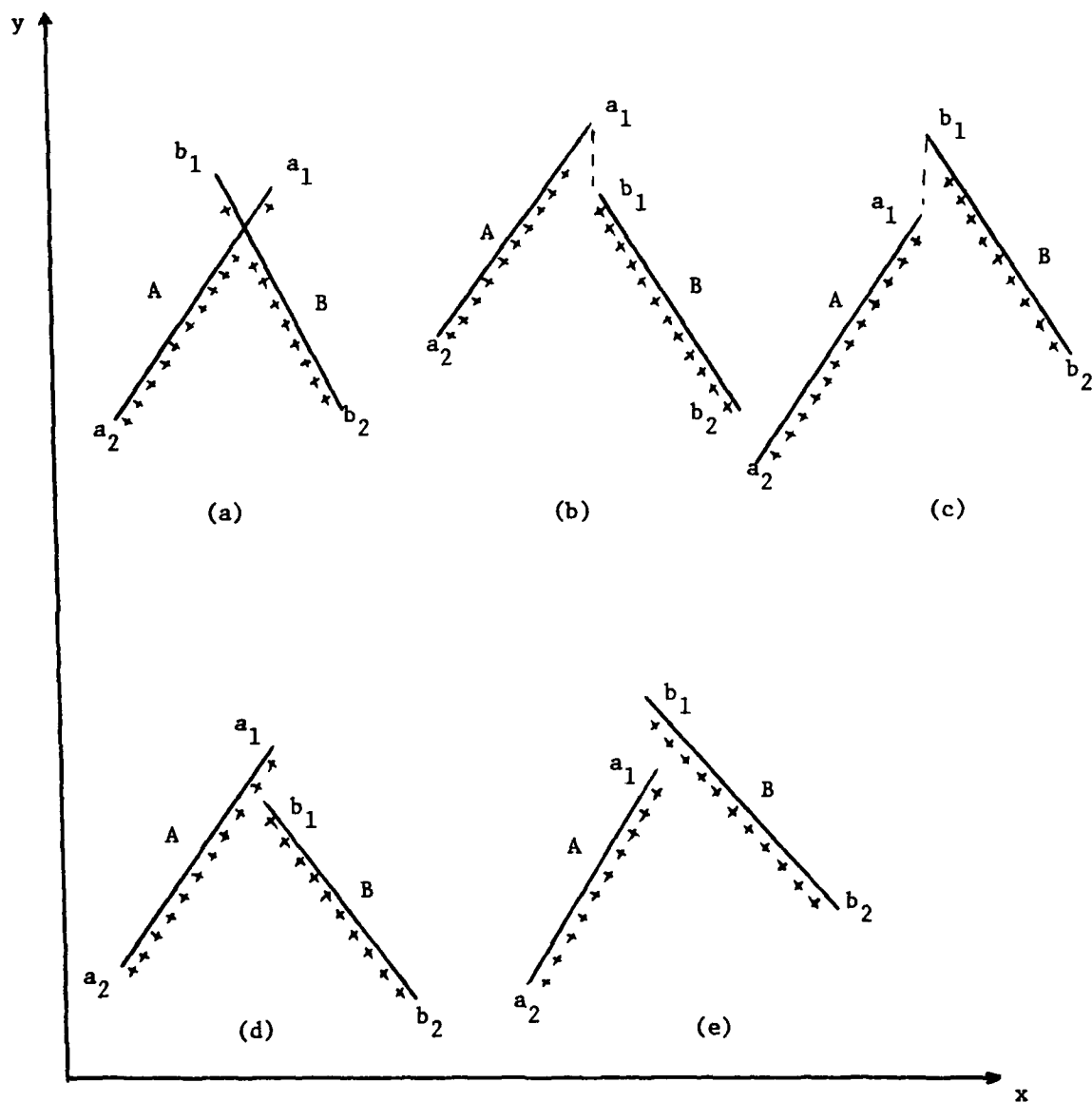
Figure 16. Configurations in Case 4: end 1 of line A
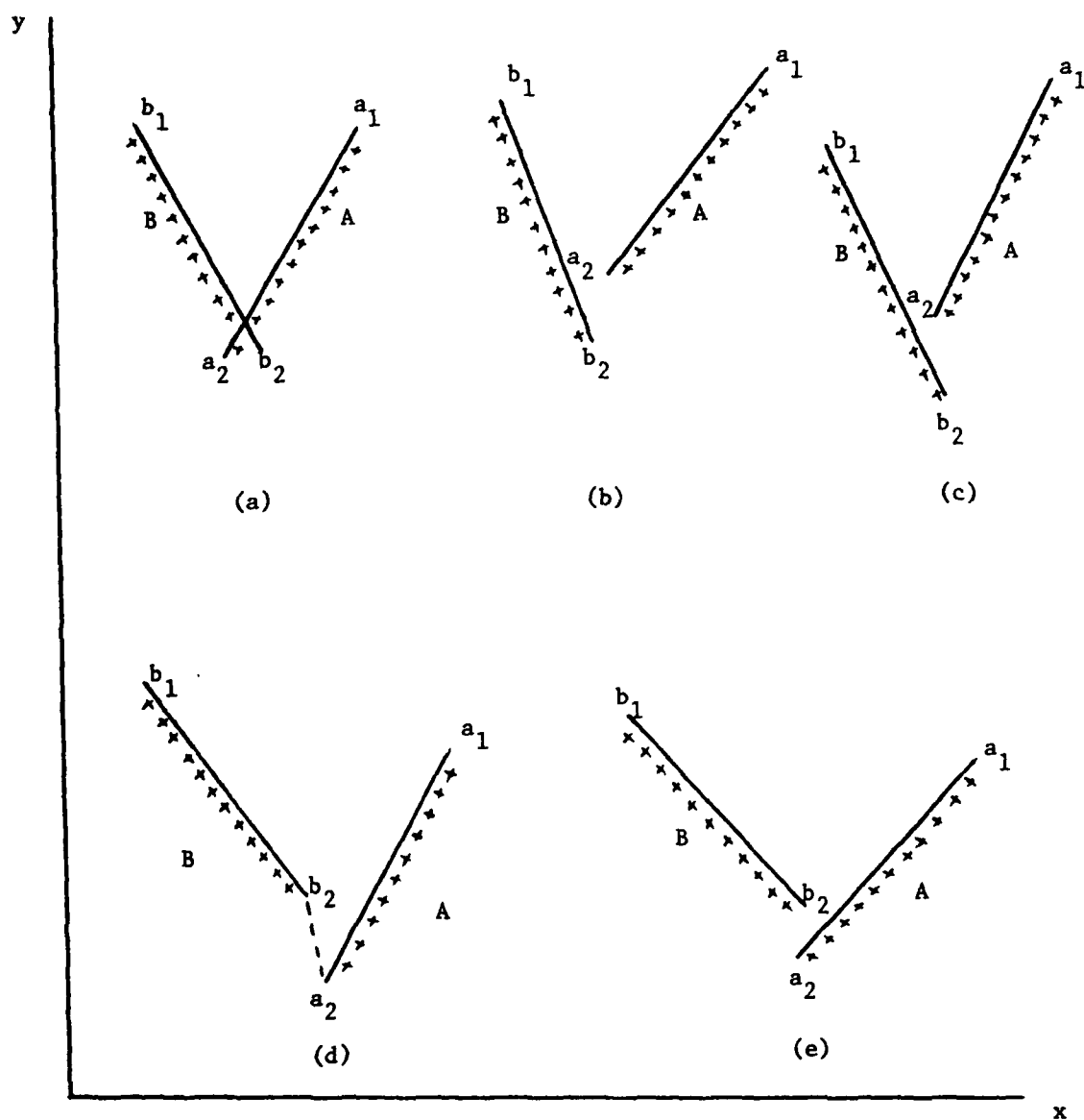
Figure 17. Configurations in Case 4: end 2 of line A.
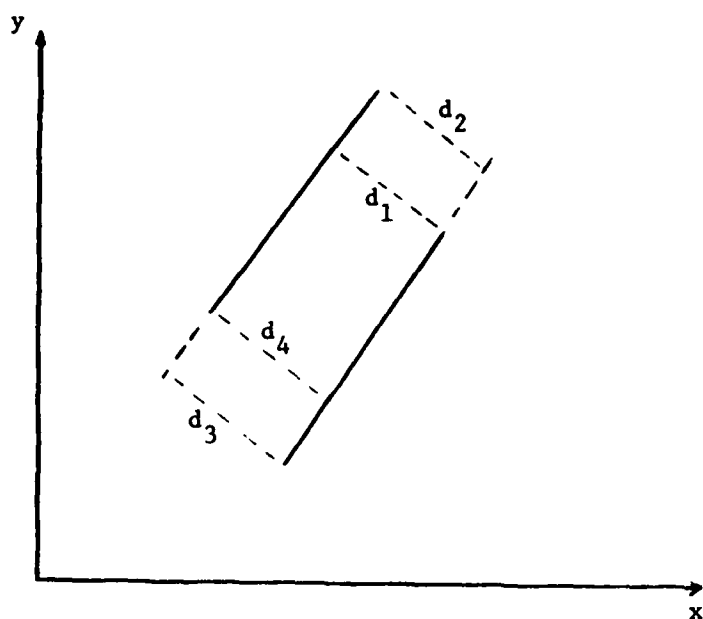
Figure 18. Calculation of the distances between two lines.

Label	Label
A	E
E	C
C	F
F	A

a) A group

b) A house
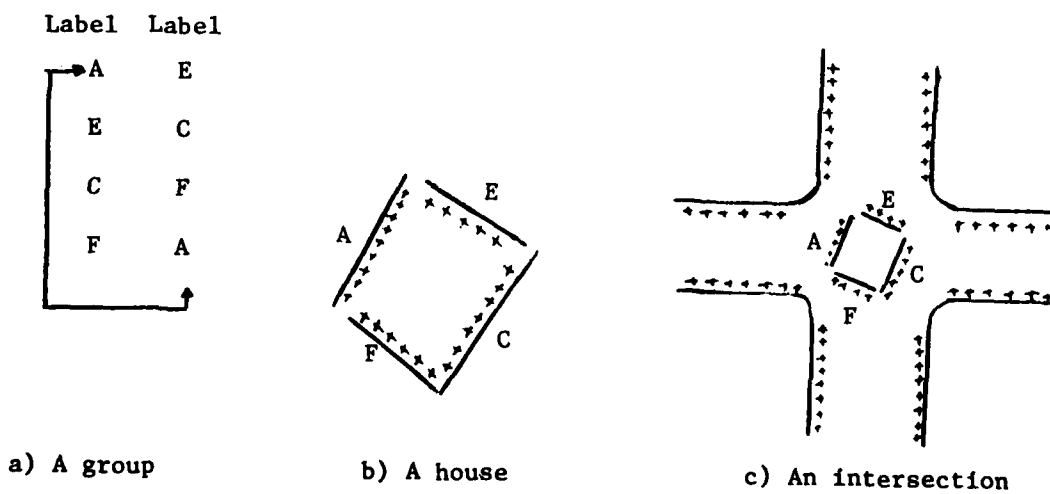
c) An intersection

Figure 19. Examples of closed groups.

Label	Label		Label	Label
A	E		E	C
E	C		C	D
C	D		E	A
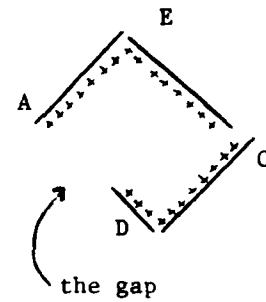
a) A group   b) A group   c) A house with missing edge.

Figure 20. Examples of semiclosed groups.
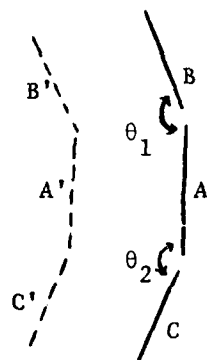
a) A curved road
$\theta_1 + \theta_2 > \theta_{min}$
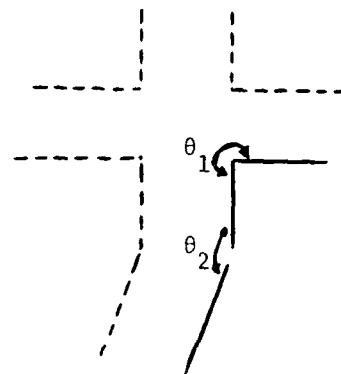
b) Three or four way intersection
$\theta_1 + \theta_2 > \theta_{min}$
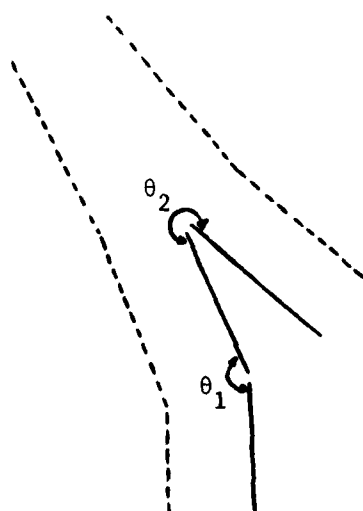
c) Merging roads
$\theta_1 + \theta_2 > \theta_{min}$
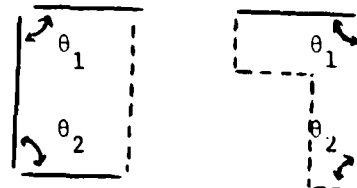
d) Single houses
$\theta_1 + \theta_2 < \theta_{min}$

Figure 21. Examples of possible cases of roads and buildings.
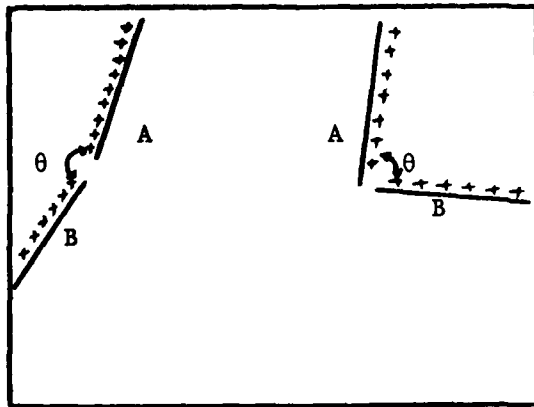
Figure 22. Case (a) of two compatible lines.
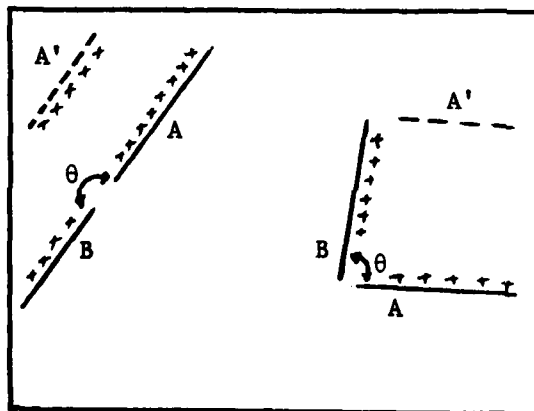
Figure 23. Case (b) of two compatible lines.
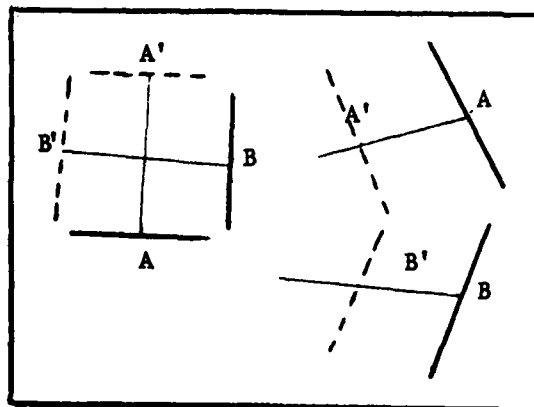
Figure 24. Case (c) of two compatible lines.
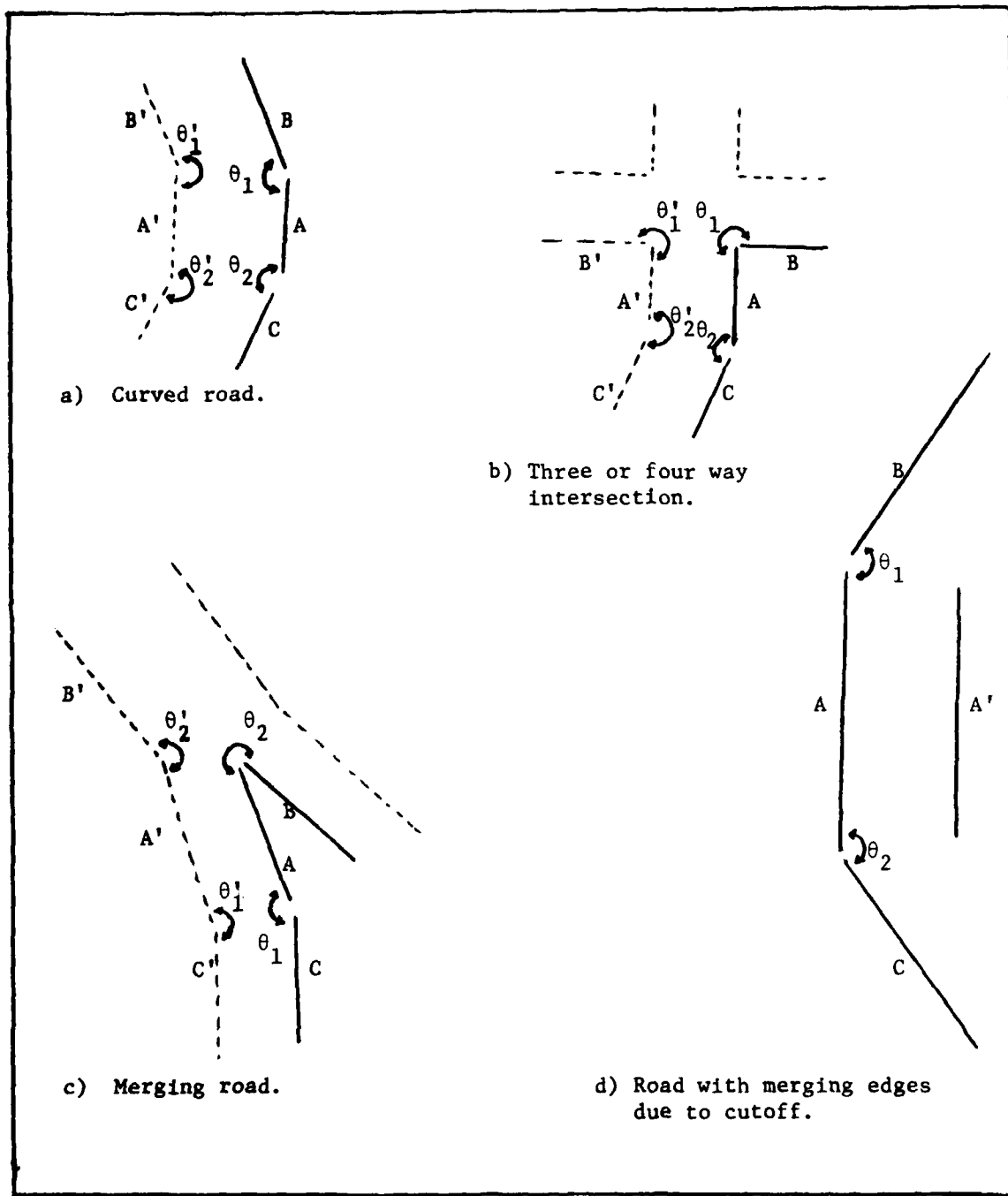
Figure 25. Examples of possible roads with high confidence.
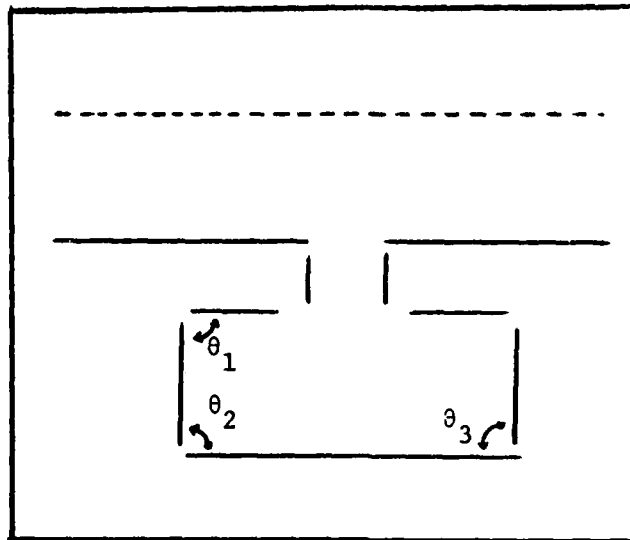
Figure 26. Example of a possible building connected to a road via a driveway (semiclosed within a group).
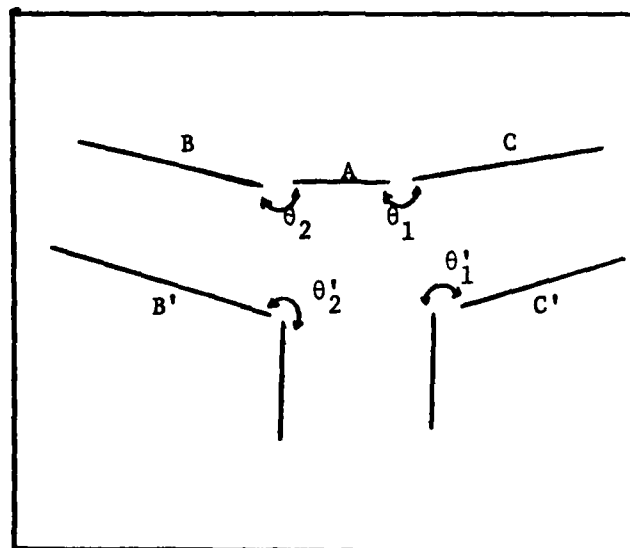
Figure 27. A special case where a line such as A has no anti-parallel, but both of its compatible neighbors are roads with high confidence.
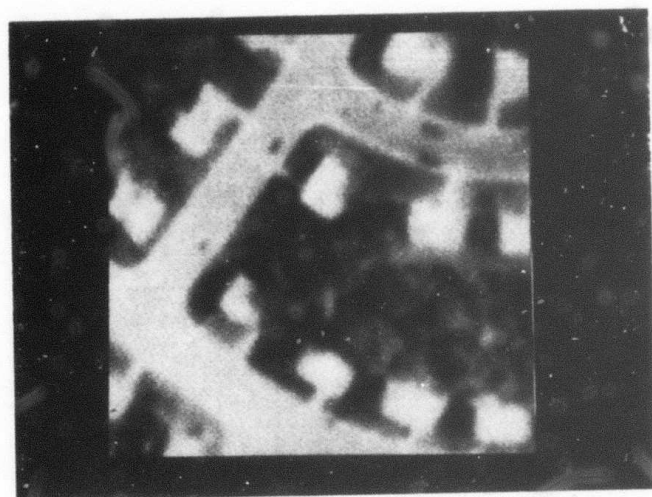
Figure 28 . A suburban scene.
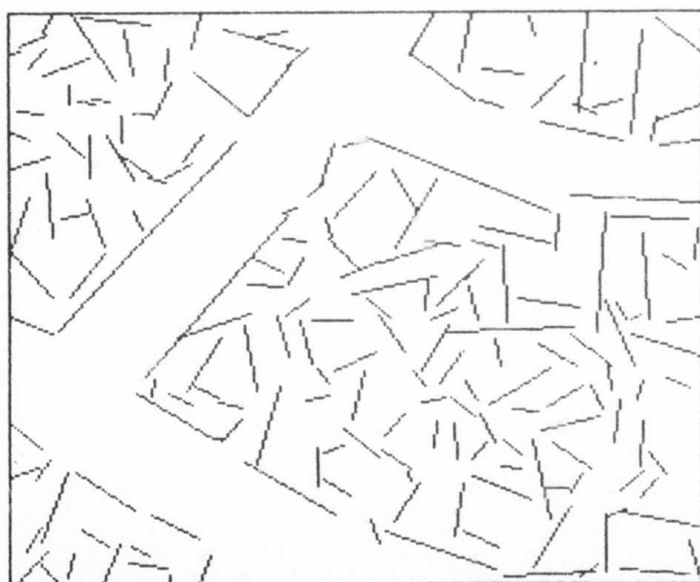
Figure 29. Line segments fitted to the edge
components of the scene of Figure 23.
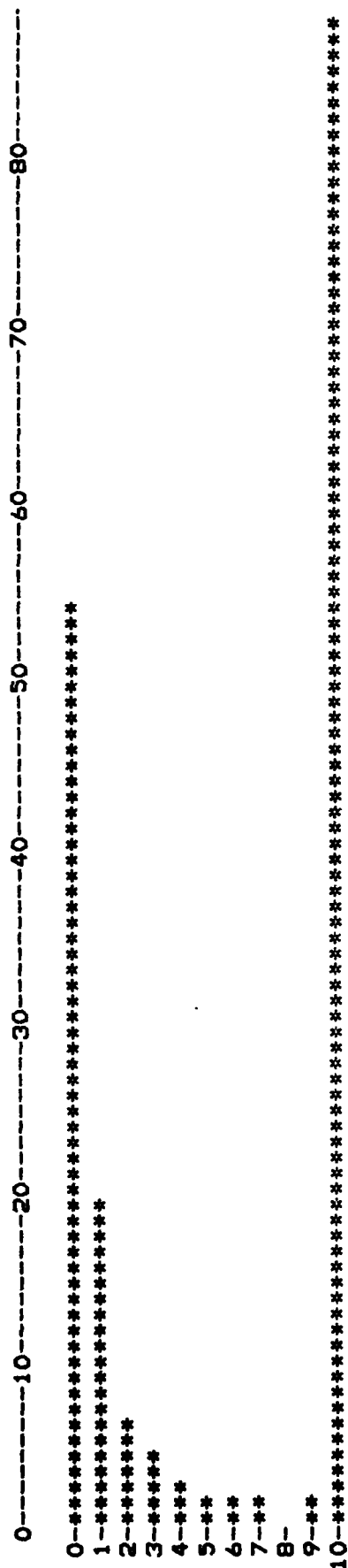
Figure 30. Histogram of the probability of "other".
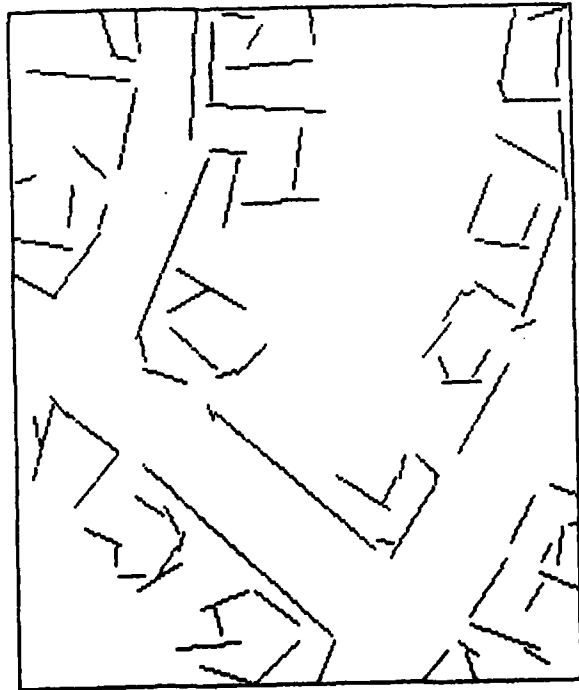
Figure 31. The line segments whose probabilities
of being a piece of a road or building
are not equal to zero.
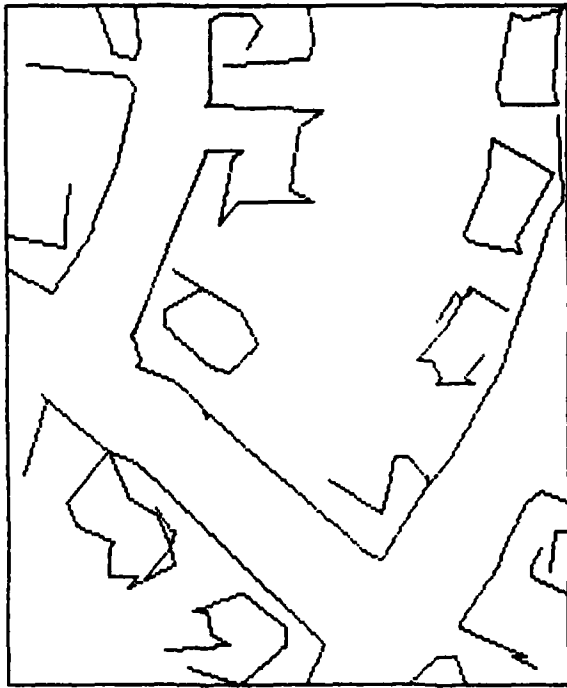
Figure 32. Results of linking compatible lines.
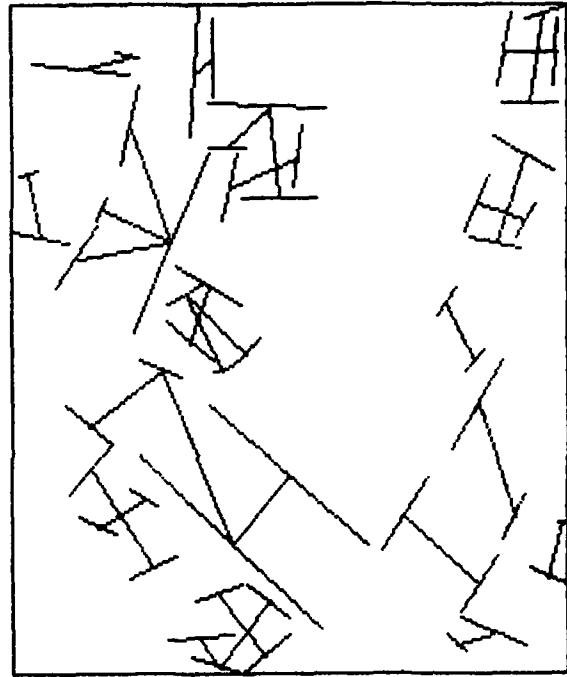
Figure 33. Antiparallel lines.
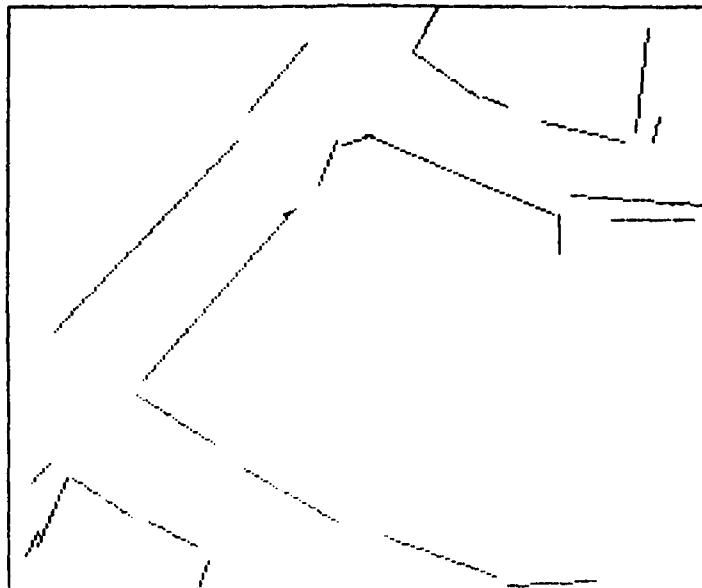
Figure 34. High confidence roads.
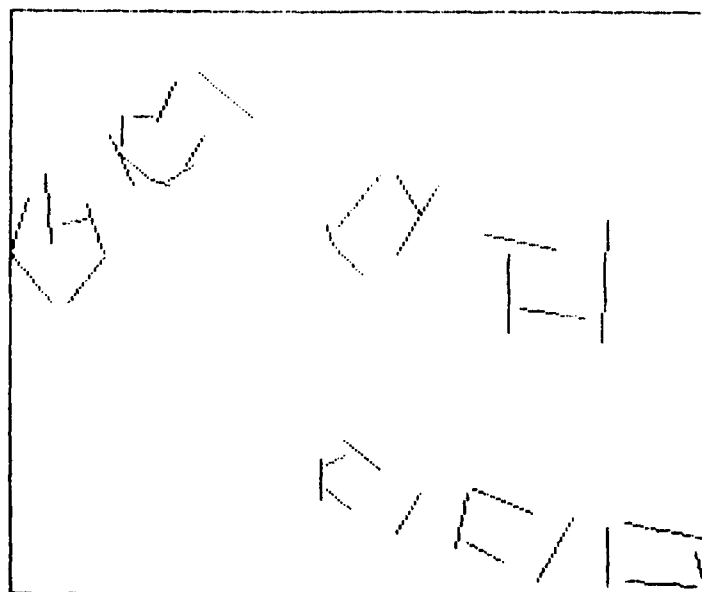
Figure 35. High confidence buildings.
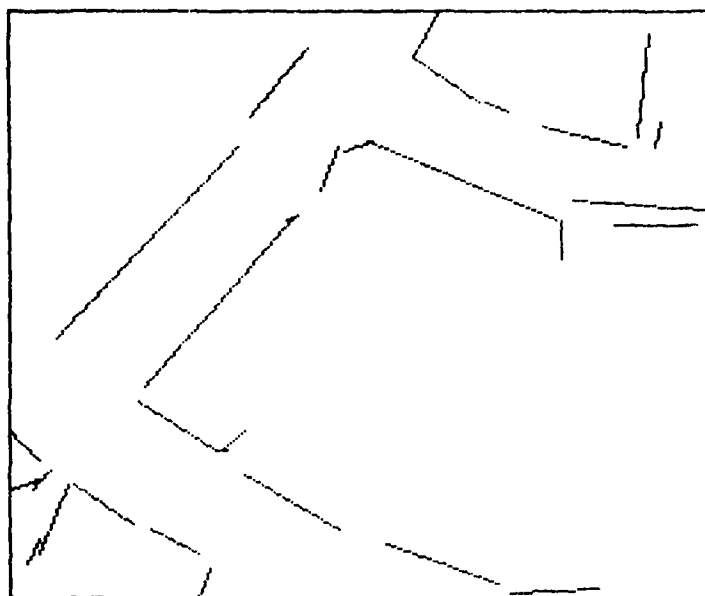
Figure 36. Roads with probability ≥ 0.75.
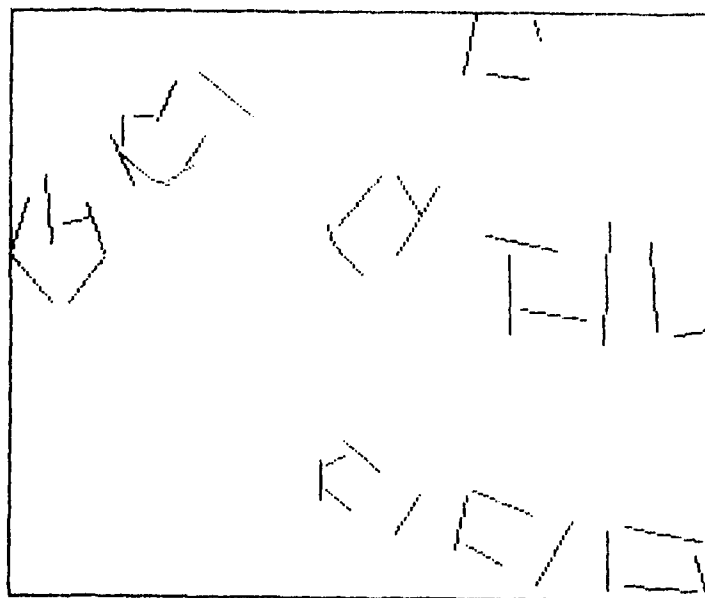
Figure 37. Buildings with probability ≥ 0.75.
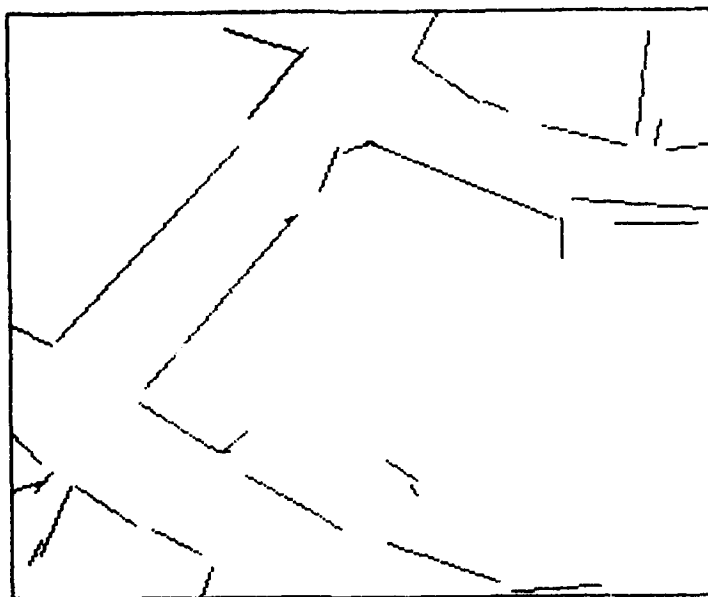
Figure 38.   Roads with probability ≥0.5.
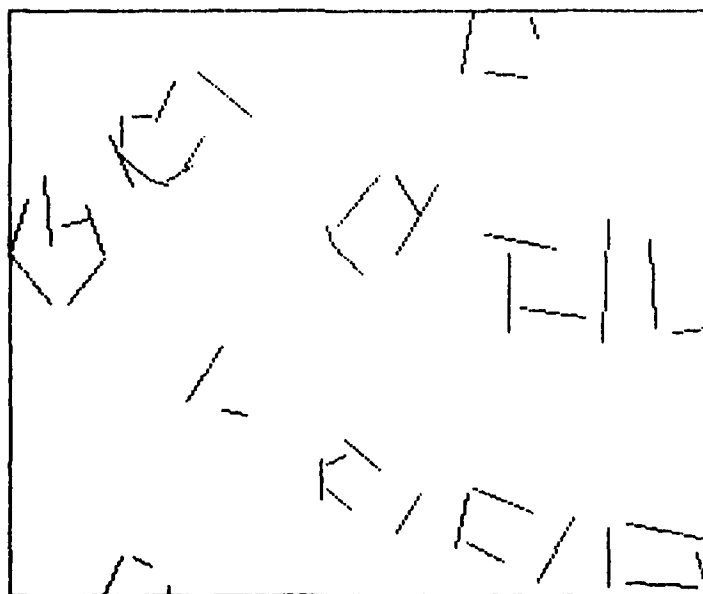
Figure 39. Buildings with probability ≥0.5.

Figure 40.   Another suburban scene
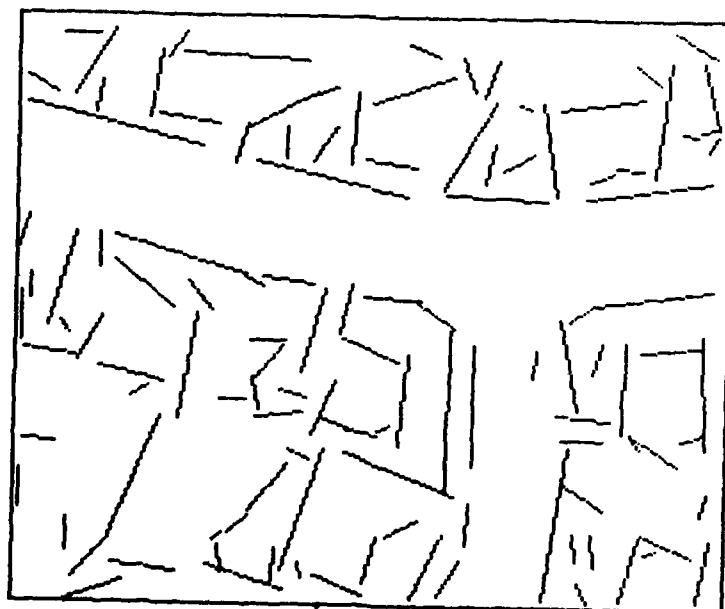
Figure 41.   Line segments fitted to the edge
components

Figure 43. Compatible lines.
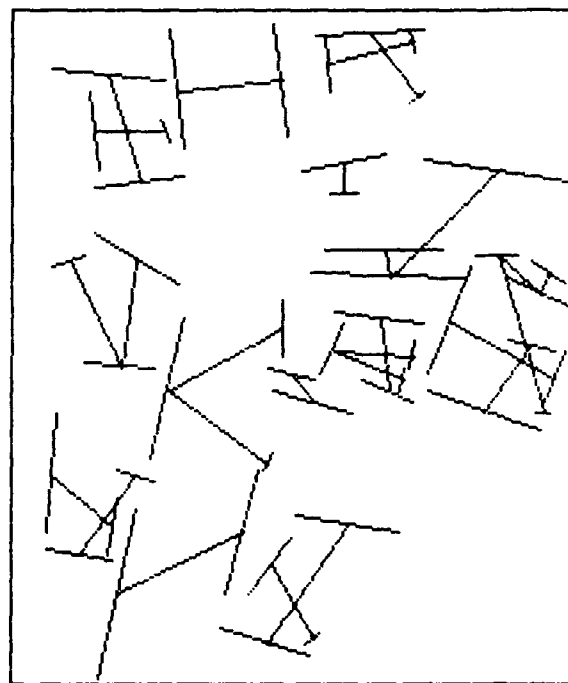
Figure 44. Antiparallel lines.

Figure 42. Line segments whose probabilities of being a piece of road or building are nonzero.
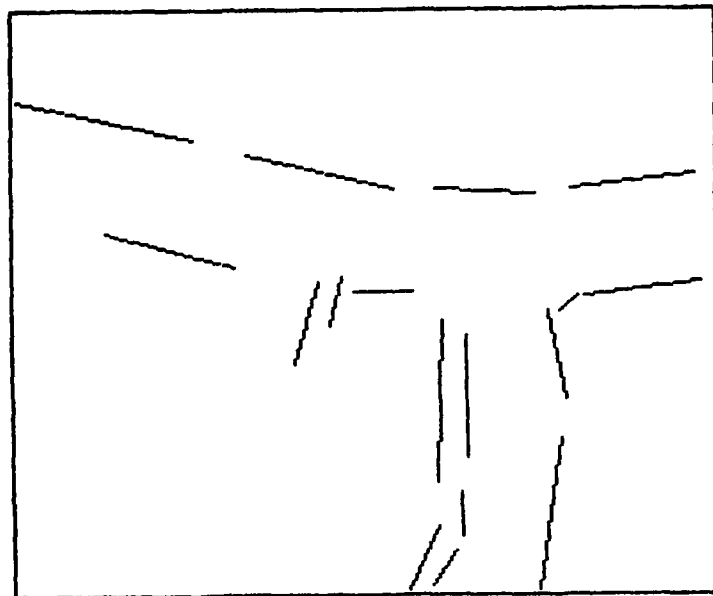
Figure 45. High confidence roads.
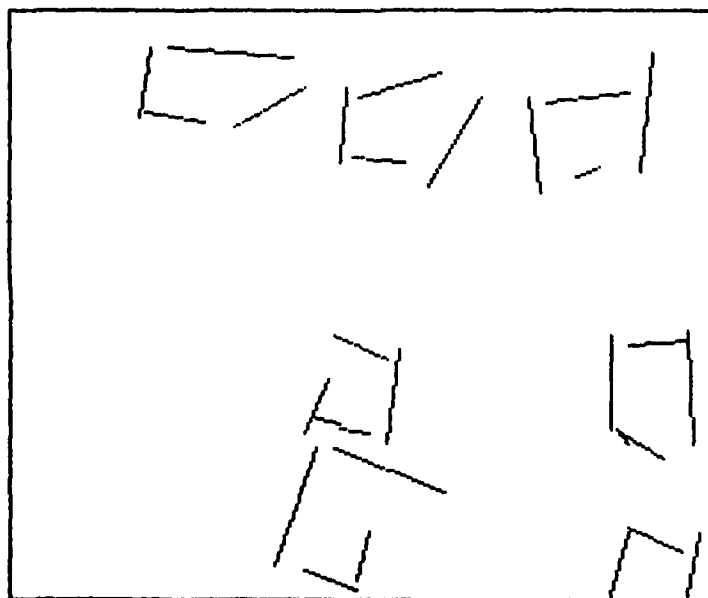
Figure 46. High confidence buildings.
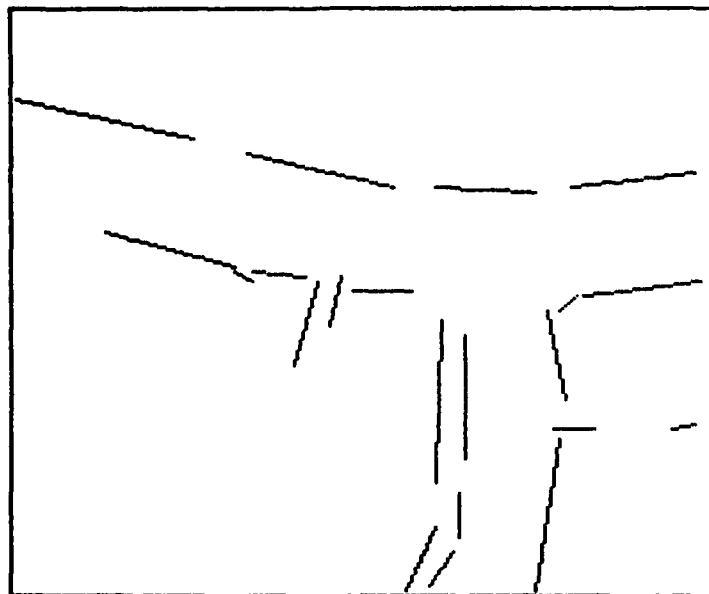
Figure 47.   Roads with probability ≥0.75.
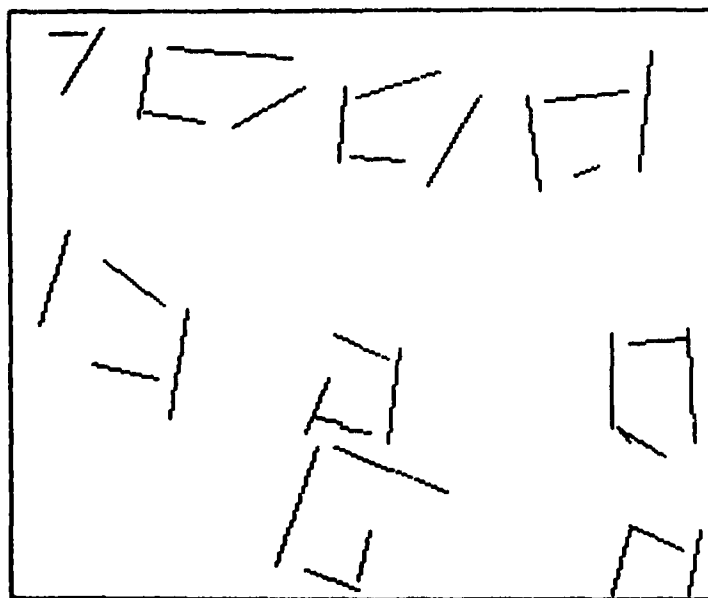
Figure 48.   Buildings with probability ≥0.75.
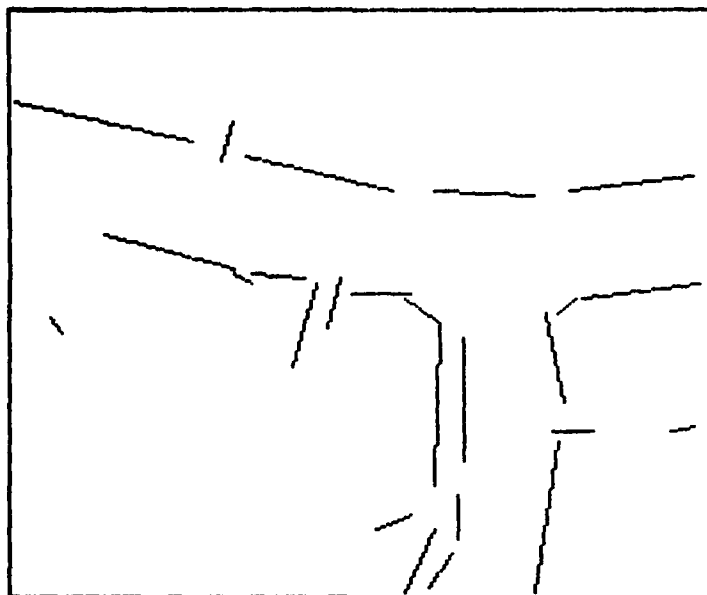
Figure 49.   Roads with probability ≥0.5.
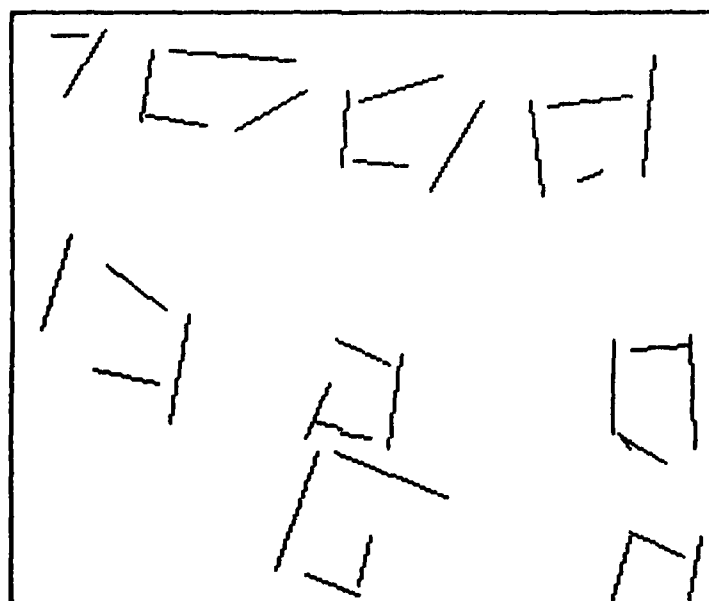
Figure 50.   Buildings with probability ≥0.5.

Figure 51. Another suburban scene.
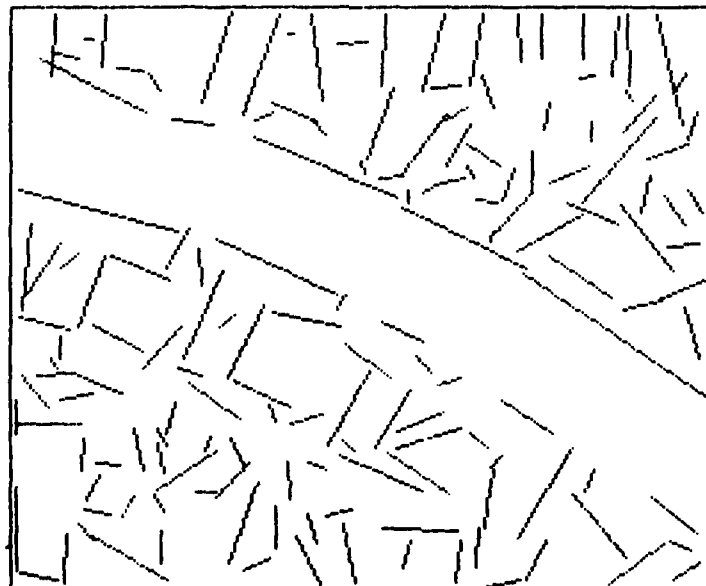
Figure 52. Line segments fitted to the edge
components.
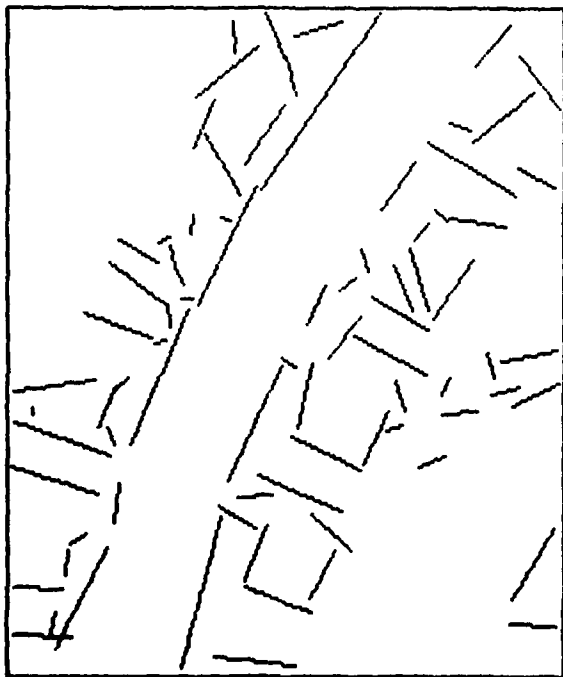
Figure 53. Line segments whose probabilities of being a piece of road or building are nonzero.
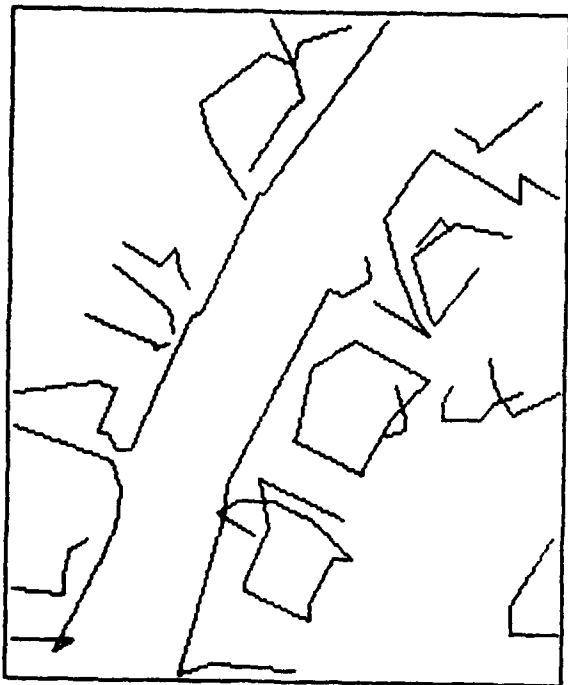
Figure 54. Compatible lines.
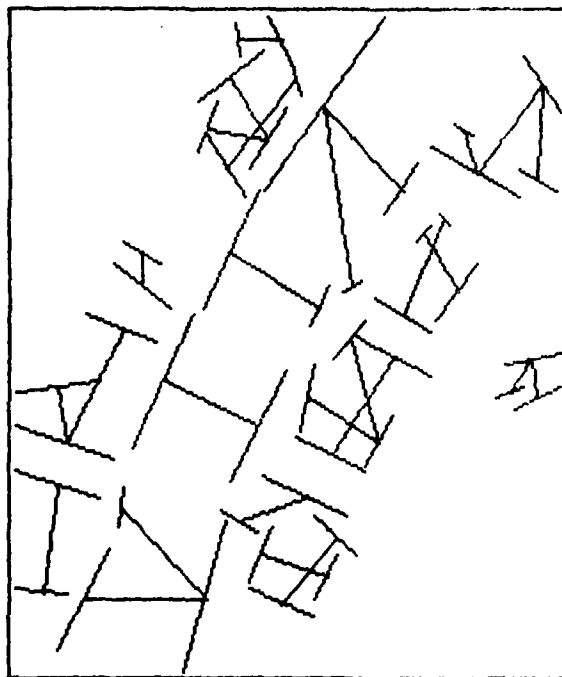
Figure 55. Antiparallel lines.
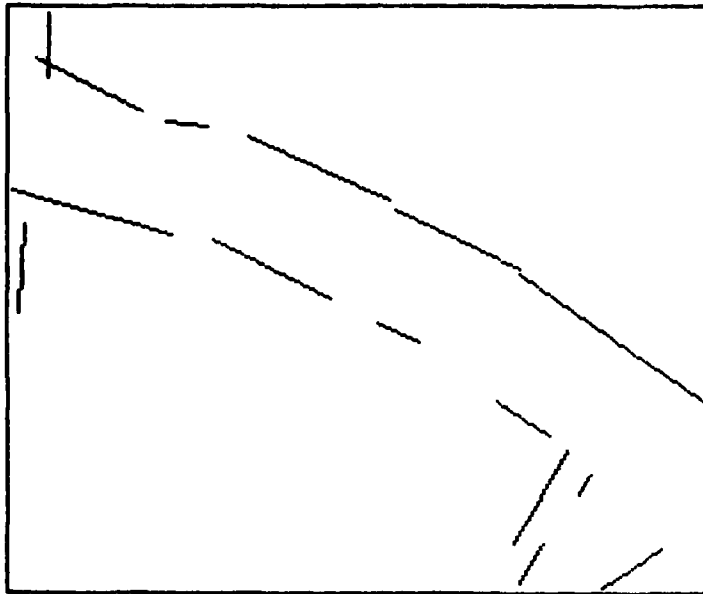
Figure 56.   High confidence roads.
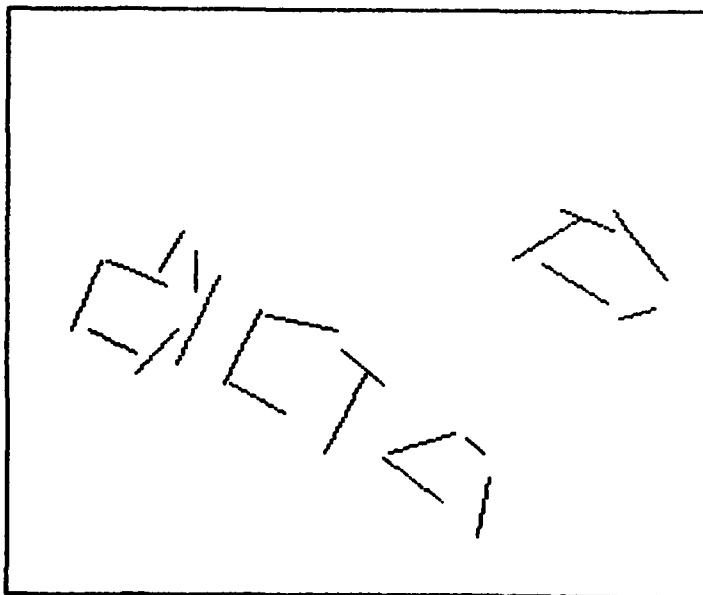
Figure 57.   High confidence buildings.
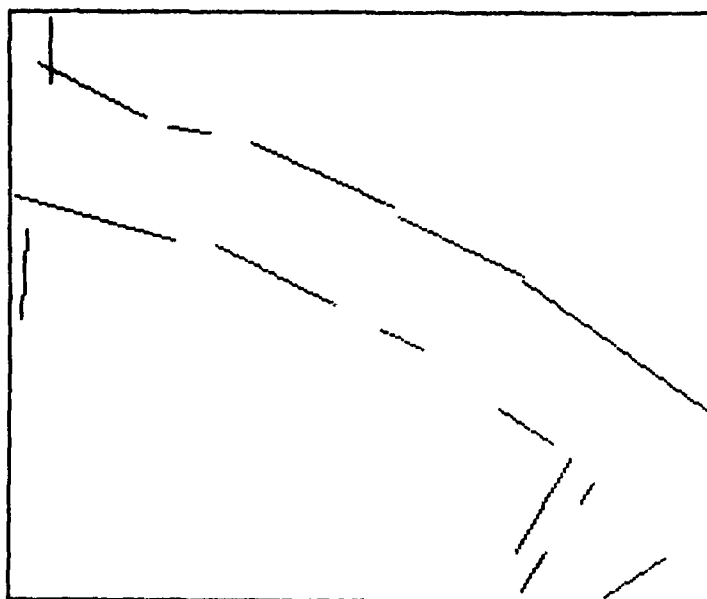
Figure 58. Road with probability ≥0.75.
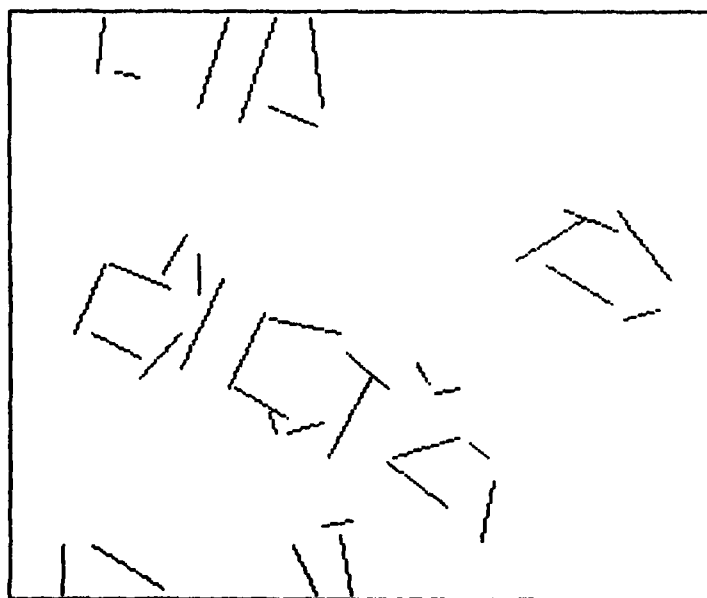
Figure 59. Buildings with probability ≥0.75.
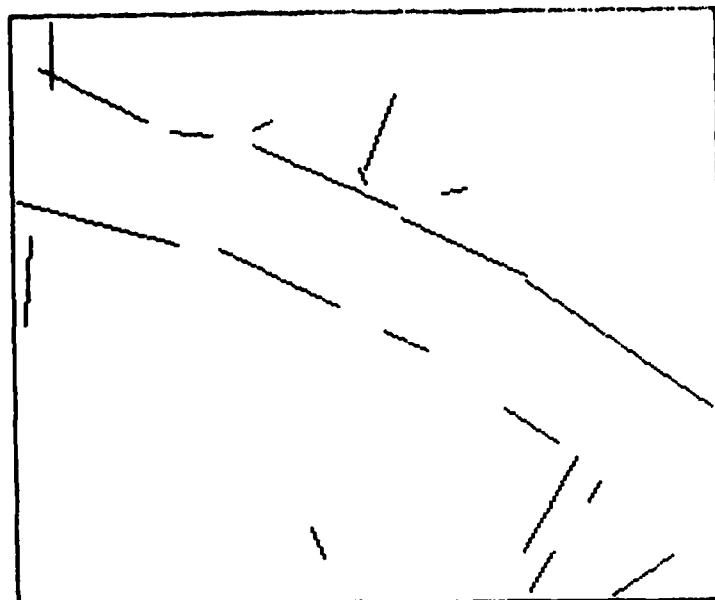
Figure 60.   Roads with probability ≥0.5.
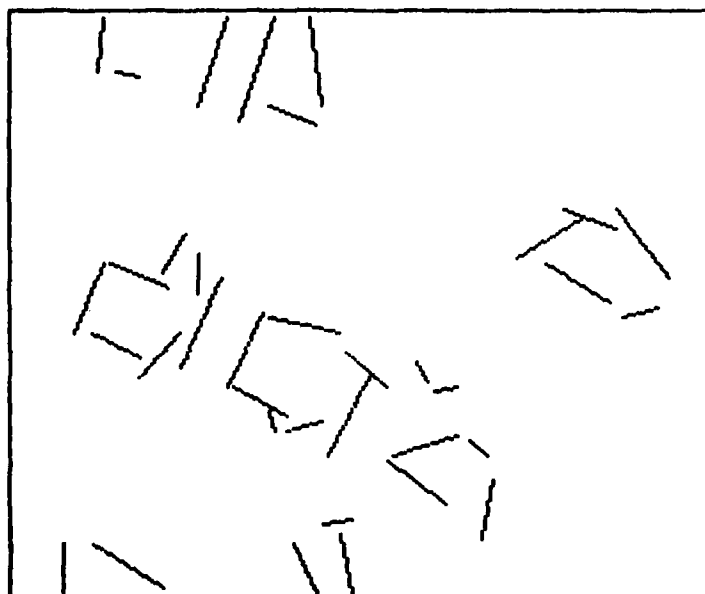
' ‑‑re 6'   Buildings with probability ≥0.5.
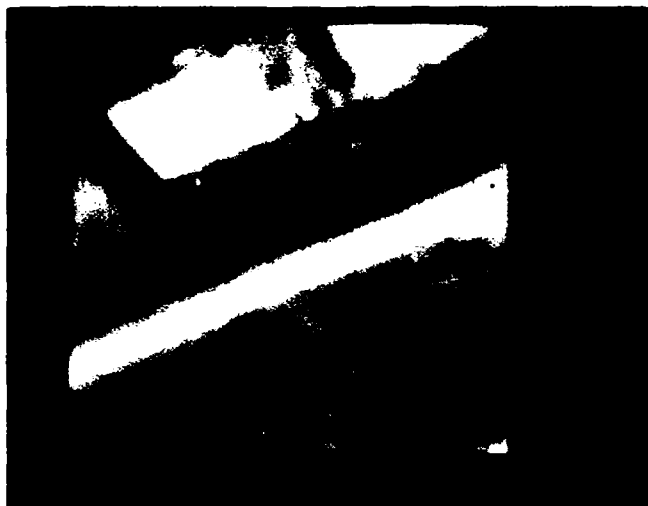
Figure 62. A non-residential scene.
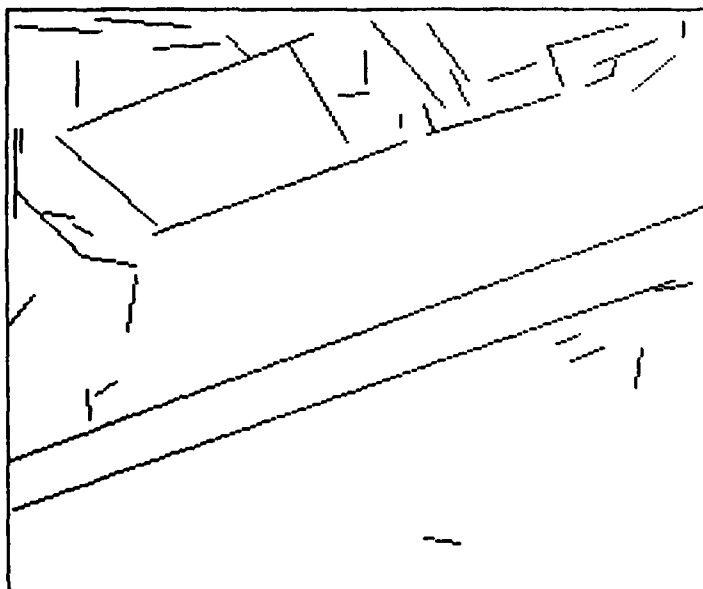
Figure 63. Line segments fitted to the edge components.
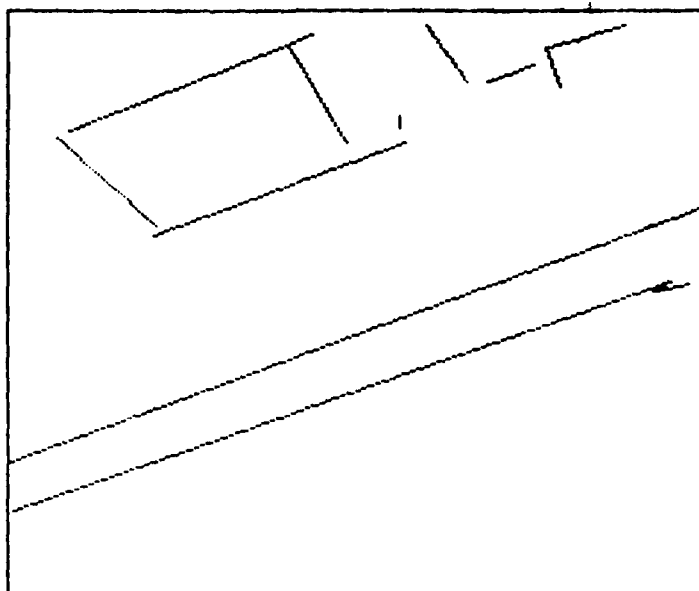
Figure 64.   Line segments whose probability of being
a piece of road or building are nonzero.
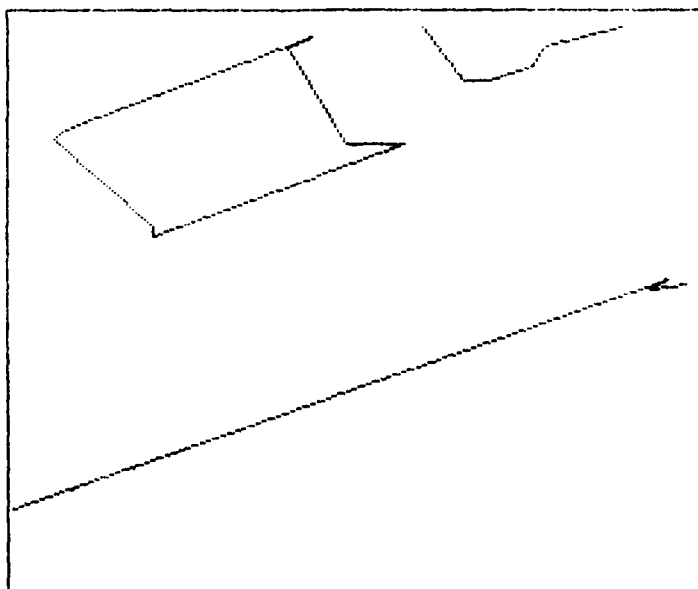
Figure 65.   Compatible lines.
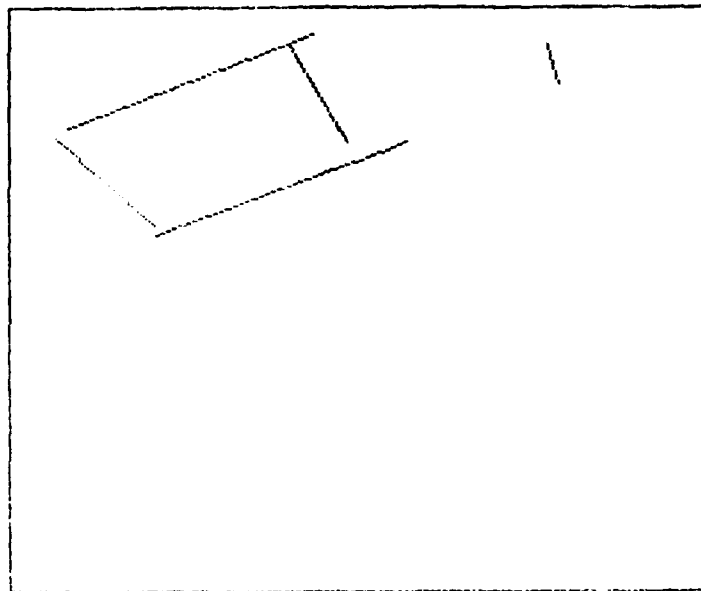
Figure 66. High confidence buildings.
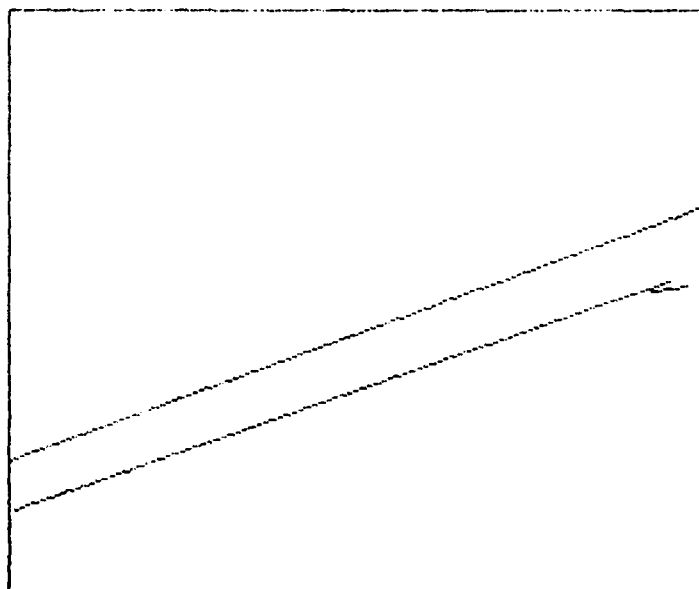
Figure 67. High confidence roads.
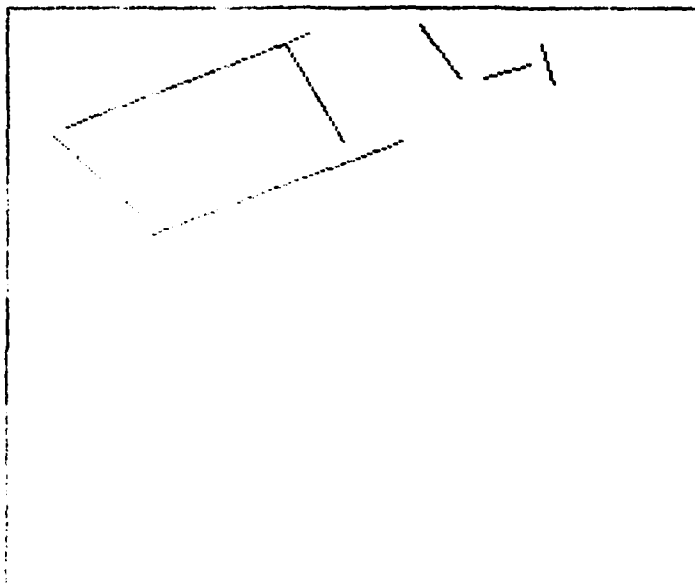
Figure 68. Buildings with probability ≥0.75 or ≥.5.
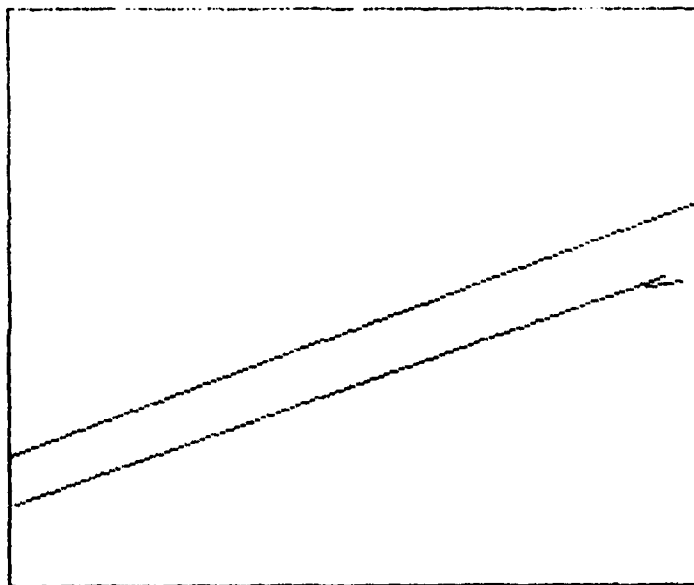
Figure 69. Roads with probability ≥0.75 or ≥.5.

Figure 70. A non-residential scene.
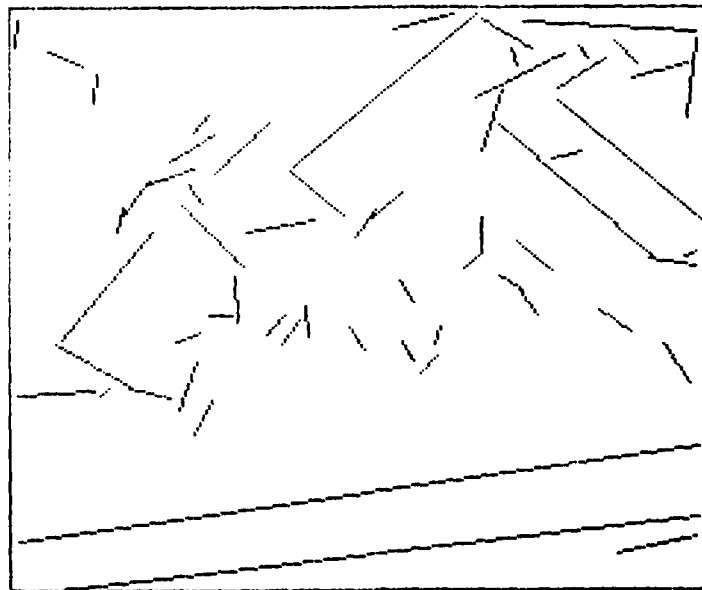
Figure 71. Line segments fitted to the edge components.
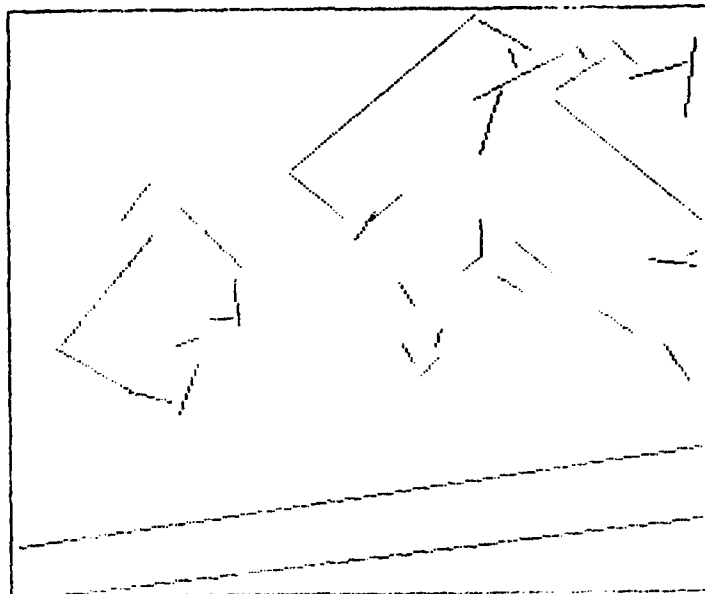
Figure 72. Line segments whose probability of being a piece of road or building are nonzero.
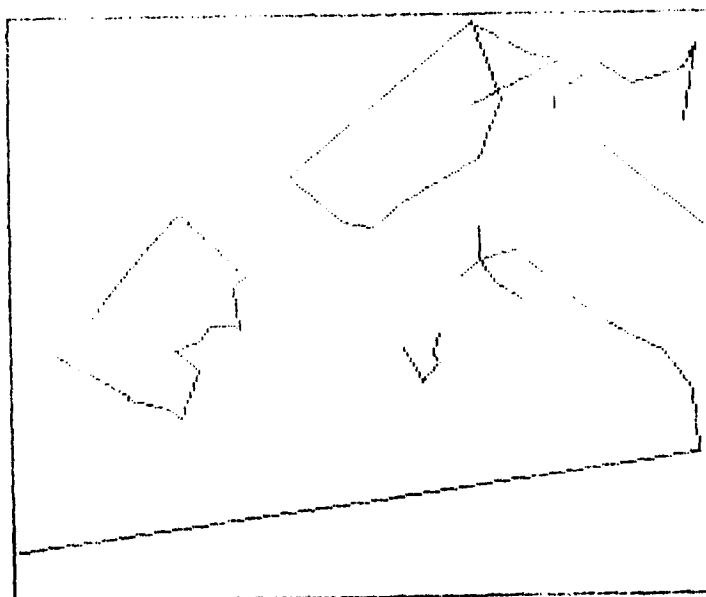
Figure 73. Compatible lines.
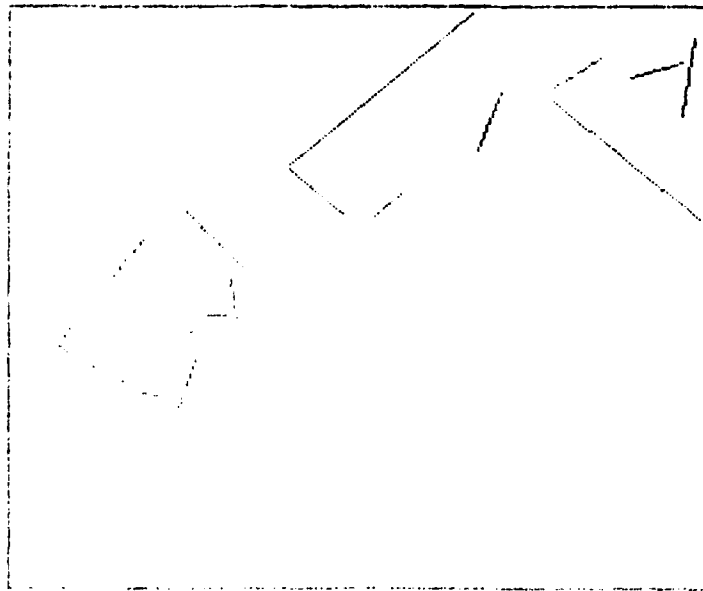
Figure 74. High confidence buildings.
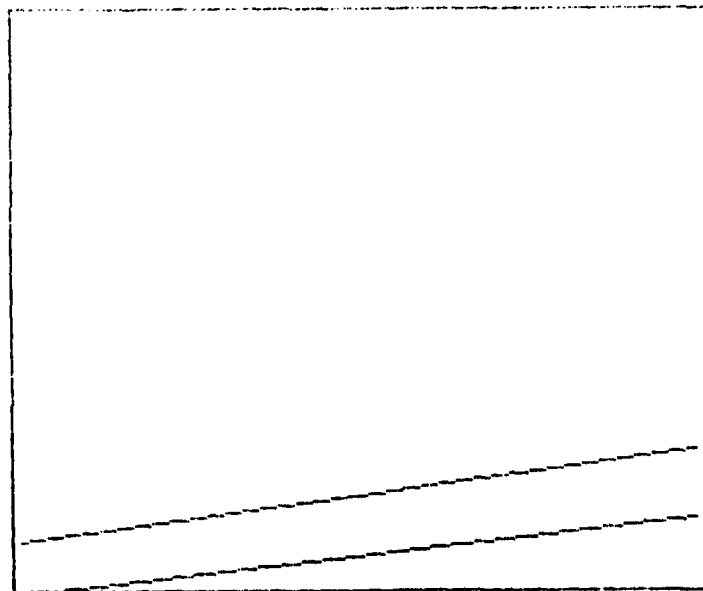
Figure 75. High confidence roads.
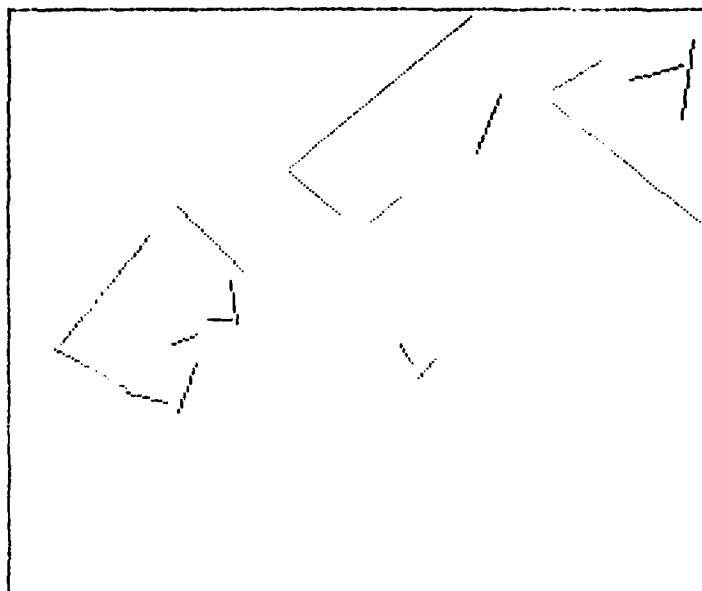
Figure 76. Buildings with probability ≥0.75 or ≥.5
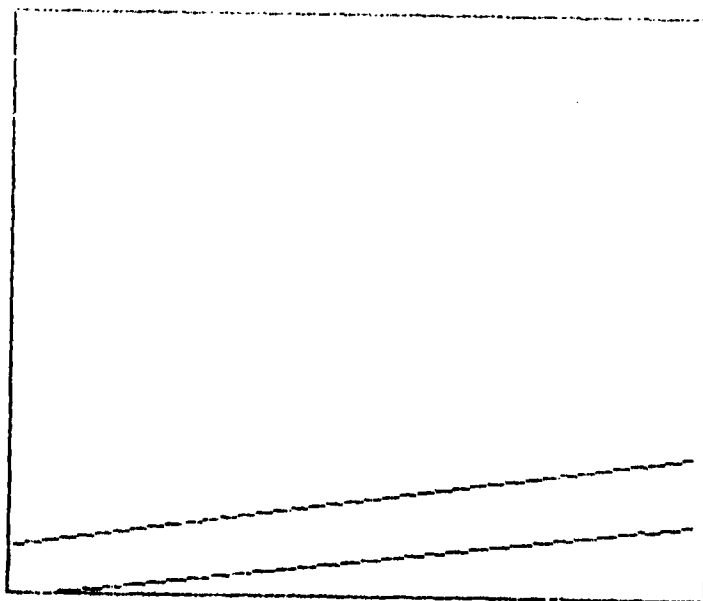
Figure 77. Roads with probability ≥0.75 or ≥ .5
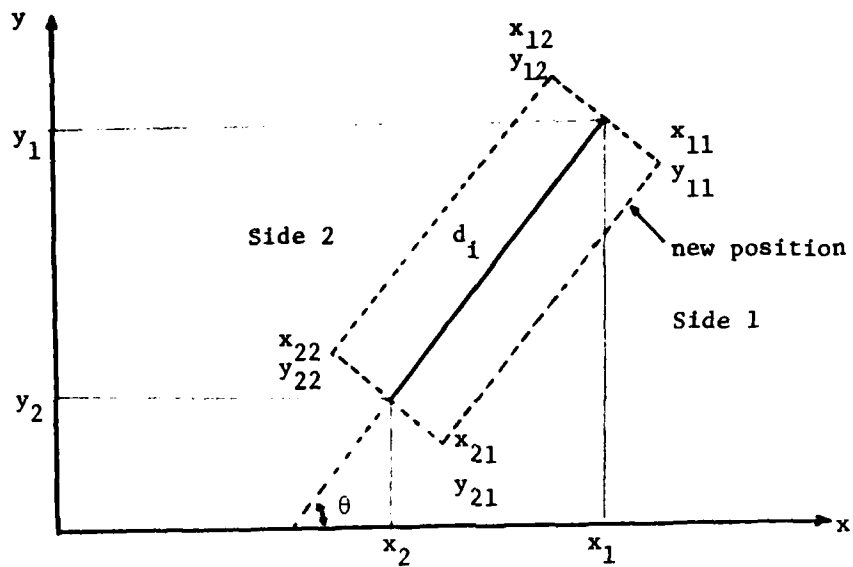
Figure 78. Displacement of the line segment.
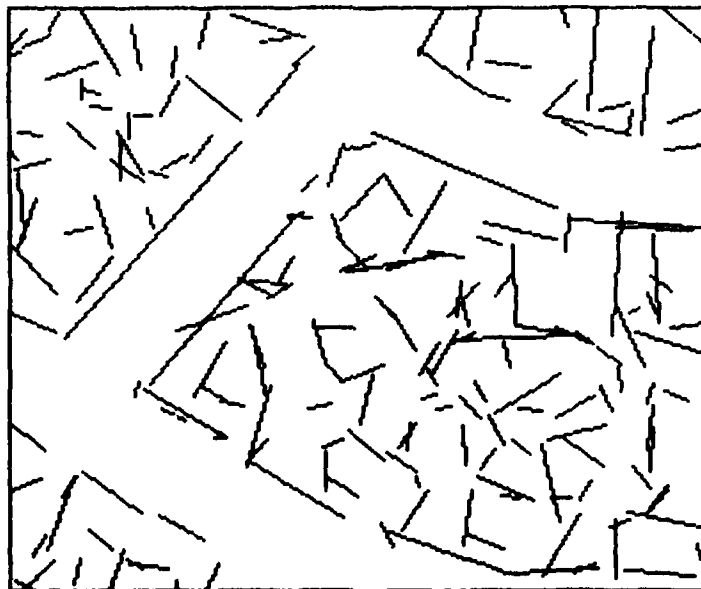
Figure 79. Relocating the lines at the maximum gradient position.
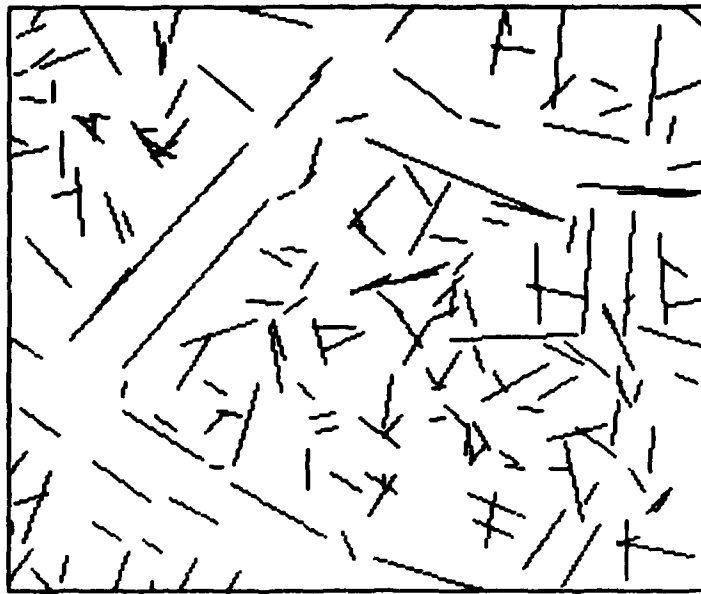
Figure 80. Relocating the lines at the minimum standard deviation position.
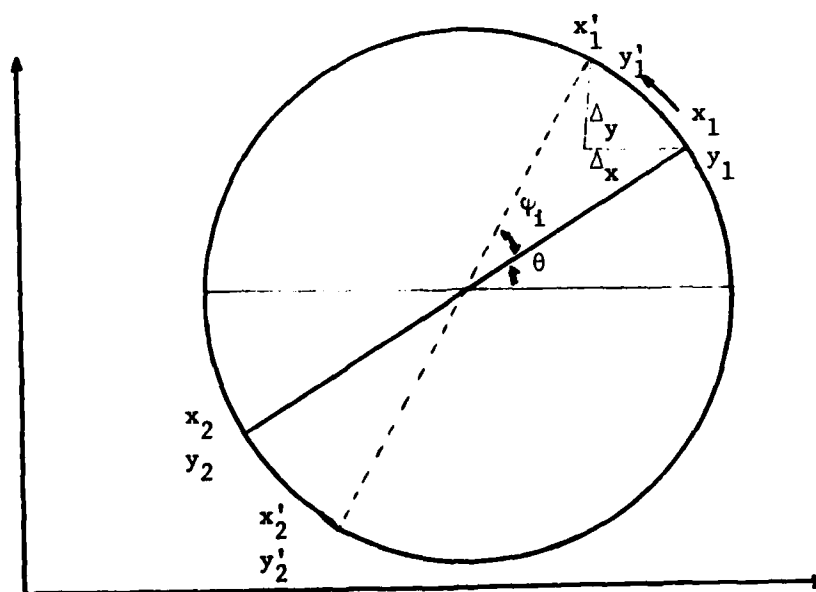
Figure 81. Changing the angle of the line segment.
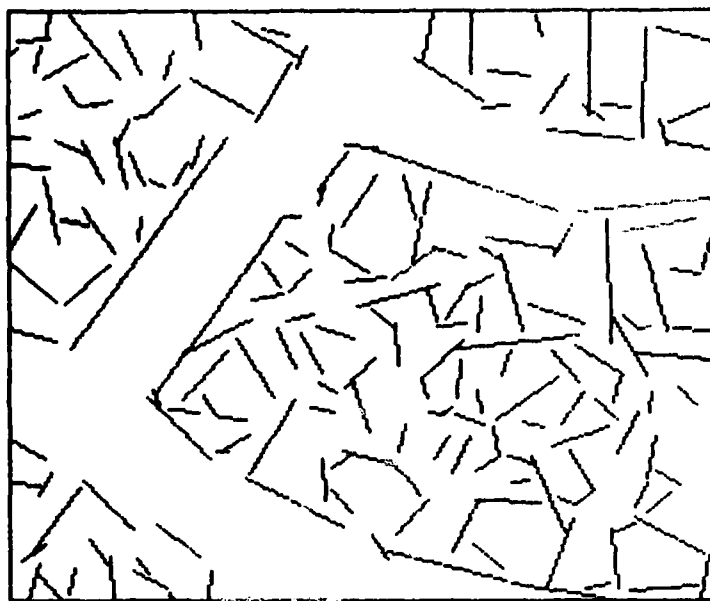
Figure 82. Rotating the lines and relocating them at the maximum gradient position.
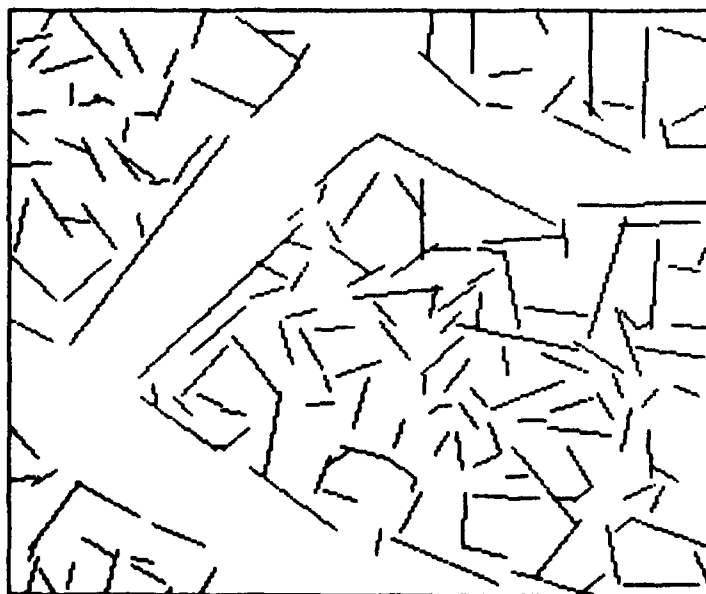
Figure 83. Rotating the lines and relocating them at the minimum standard deviation position.
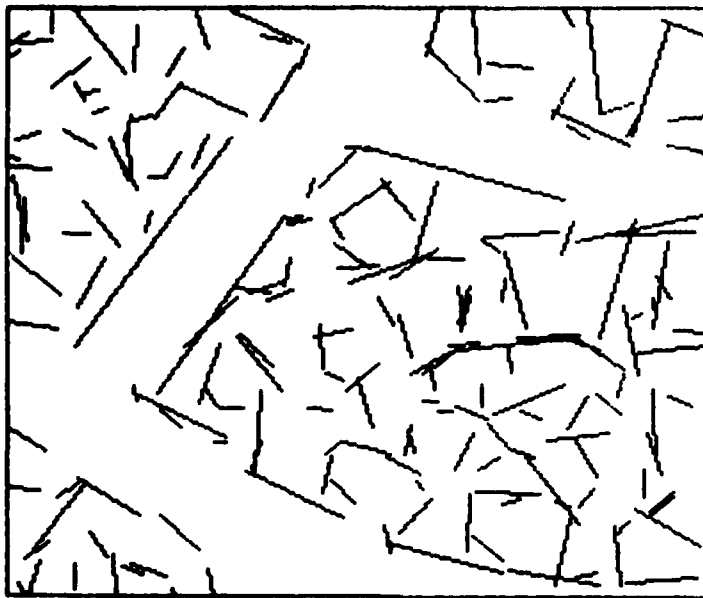
Figure 84. First translation then rotation and relocating at the maximum gradient position.

Figure 85. First translation then rotation and relocating at the minimum standard deviation position.

Figure 86. First rotation then translation and relocating at the maximum gradient position.
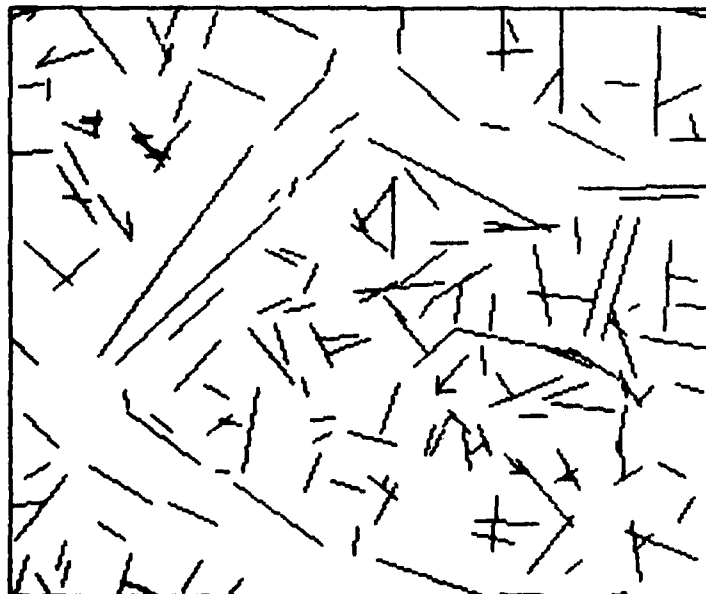
Figure 87. First rotation then translation and relocating at the minimum standard deviation position.
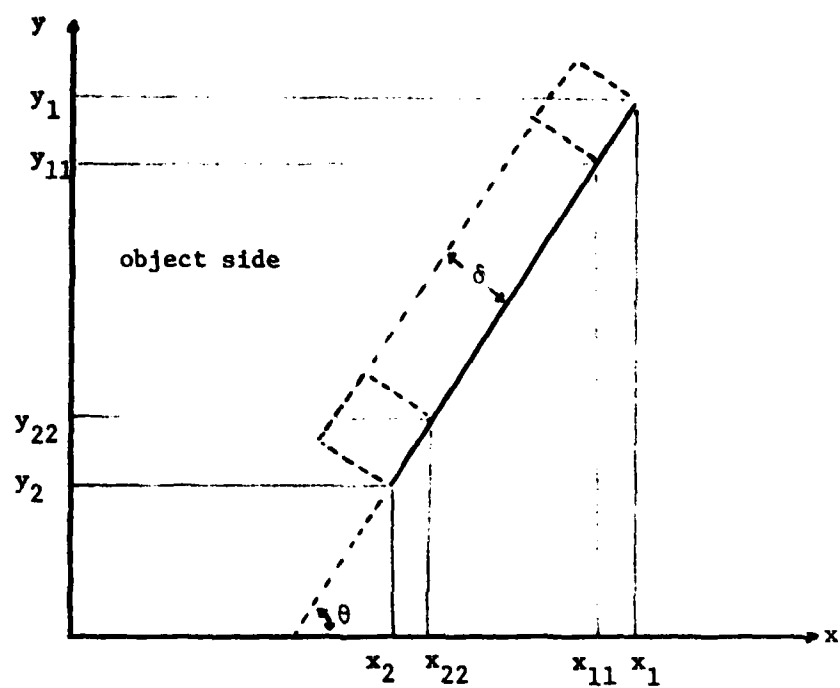
Figure 88. Length adjustment.

Figure 89.   Threshold = 5 in length adjustment.

Figure 90.   Threshold = 4 in length adjustment.

Figure 91. Threshold = 3 in length adjustment.
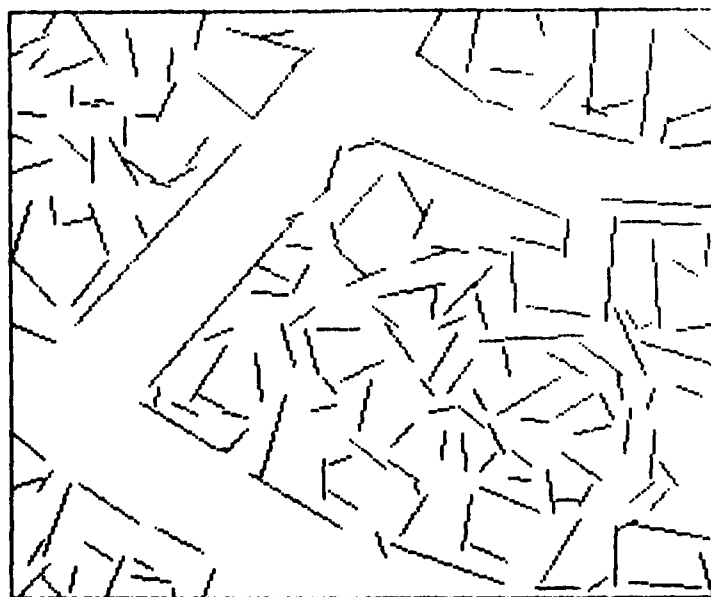
Figure 92. Length adjustment with threshold =
standard deviation of the gray level
on the object side of the line segment.
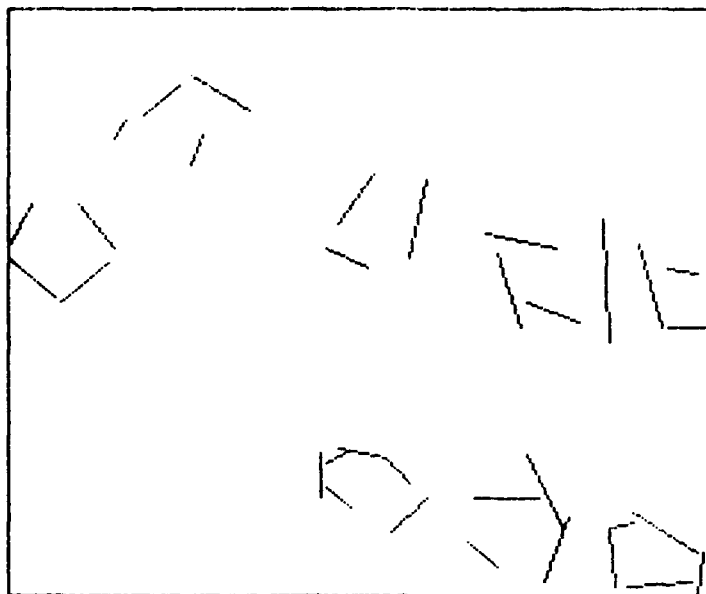
Figure 93. Buildings with confidence 1 after maximum gradient angle adjustment.
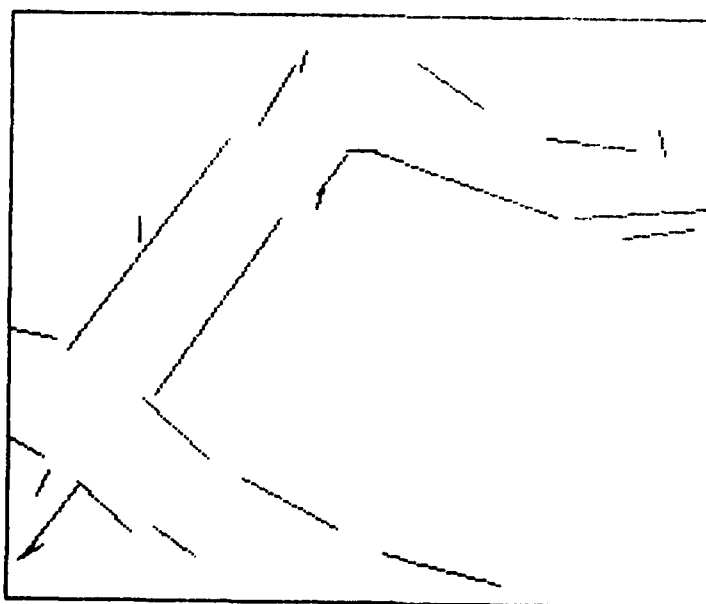
Figure 94. Roads with confidence 1 after maximum gradient angle adjustment.
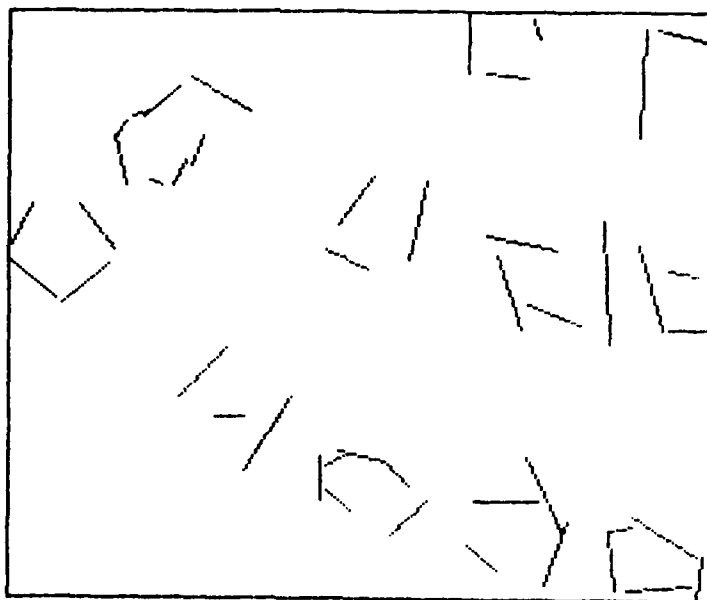
Figure 95. Buildings with *confidence* .75 after maximum gradient angle
adjustment.
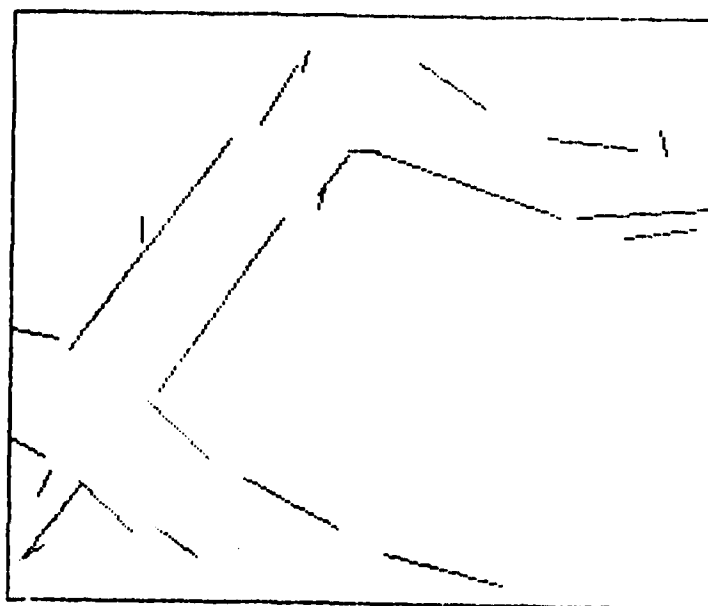
Figure 96. Roads with confidence .75 after maximum gradient angle
adjustment.
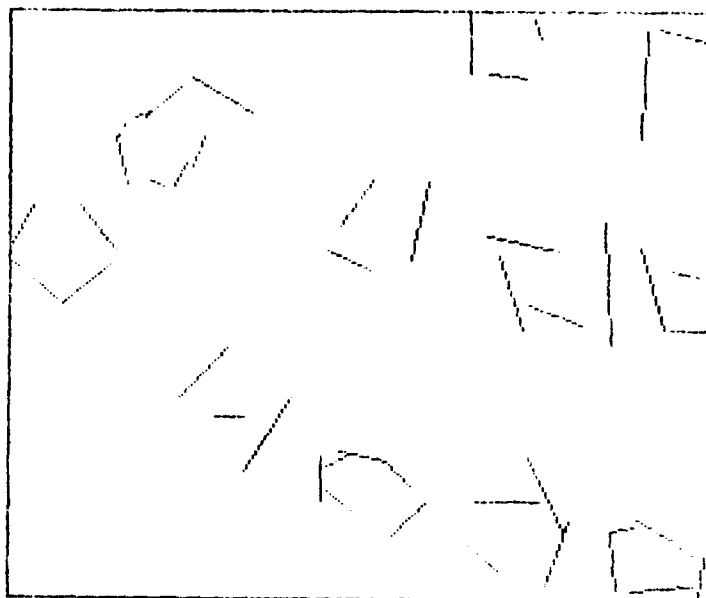
Figure 97. Buildings with confidence .5 after maximum gradient angle adjustment.
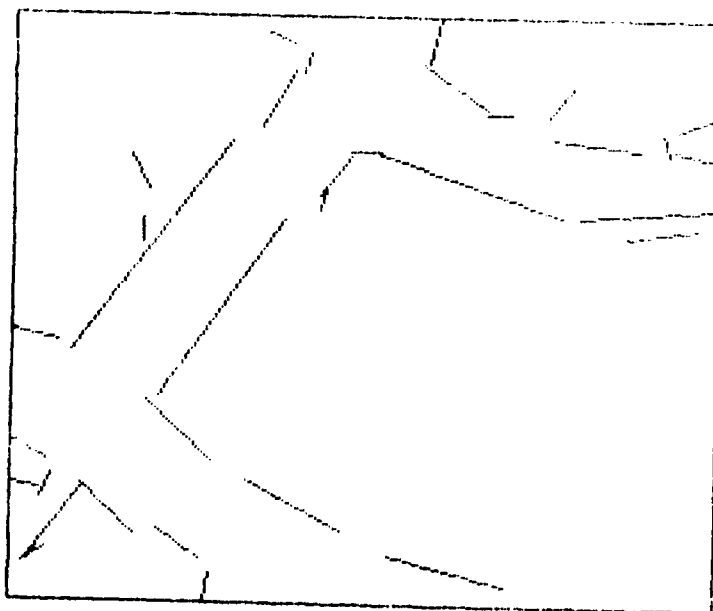
Figure 98. Roads with confidence .5 after maximum gradient angle adjustment.
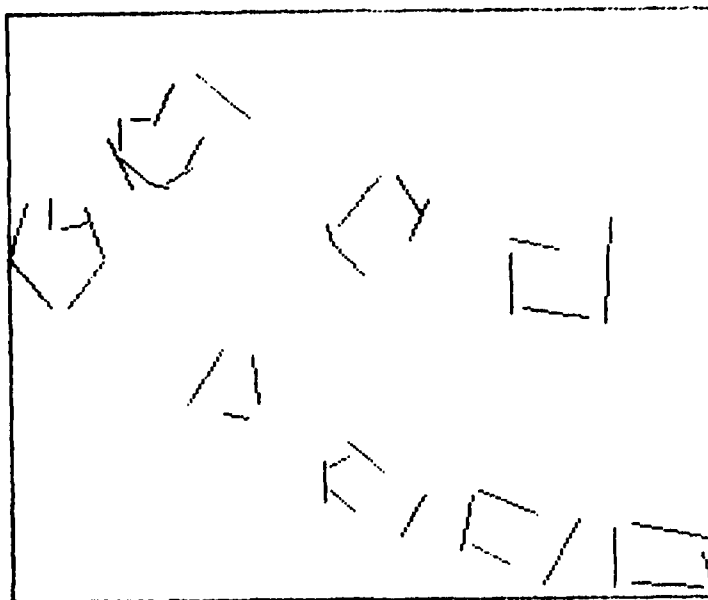
Figure 99. Buildings with confidence 1 after dynamic-threshold length adjustment.
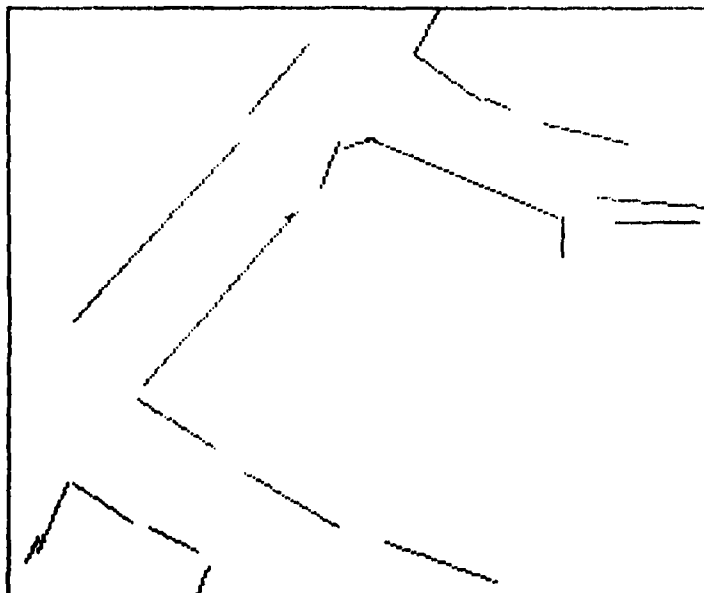
Figure 100. Roads with confidence 1 after dynamic-threshold length adjustment.
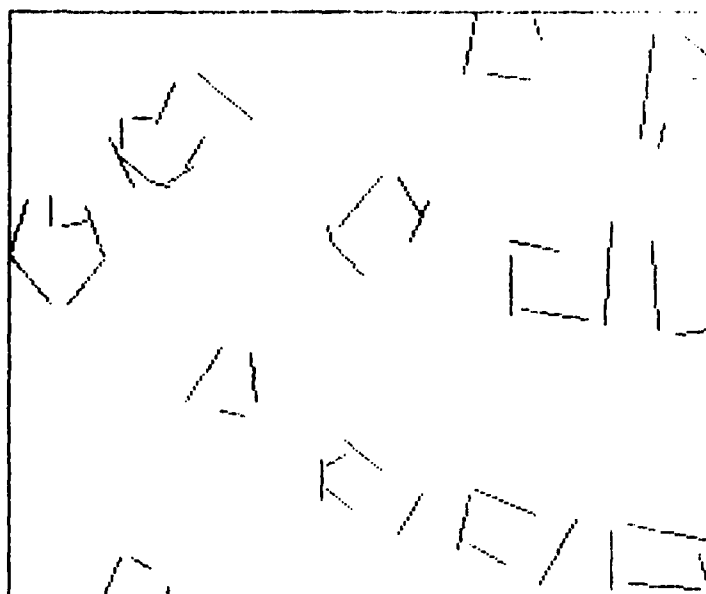
Figure 101. Buildings with confidence .75 after dynamic-threshold
             length adjustment.
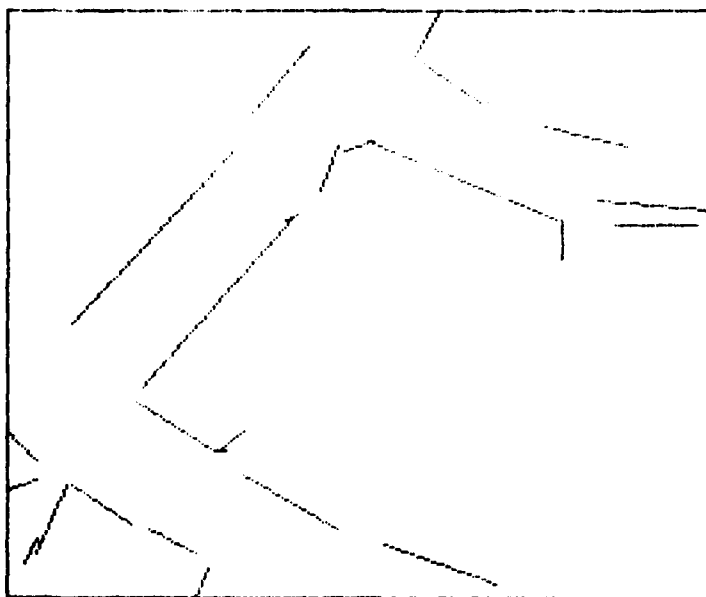
Figure 102. Roads with confidence .75 after dynamic-threshold
             length adjustment.
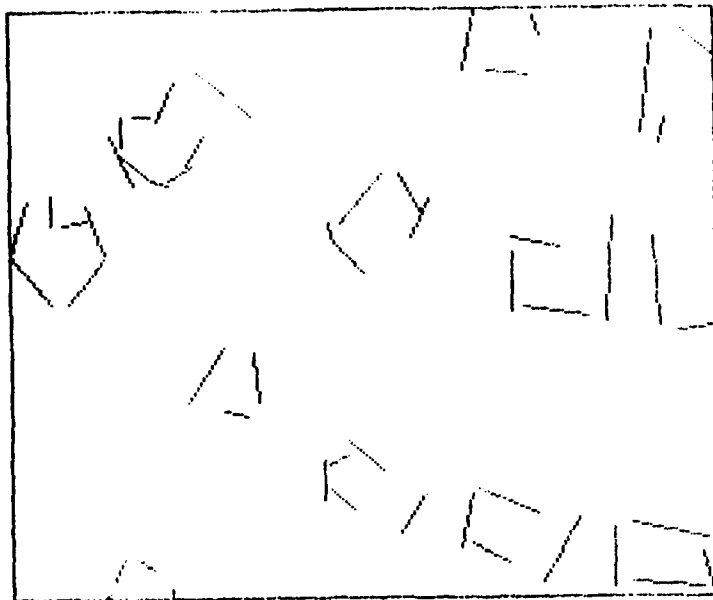
Figure 103. Buildings with confidence .5 after dynamic-threshold
length adjustment.
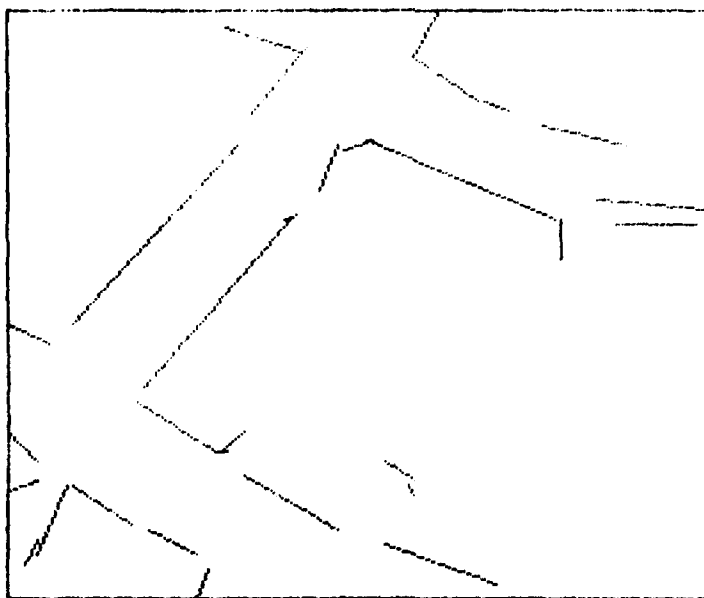
Figure 104. Roads with confidence .5 after dynamic-threshold
length adjustment.
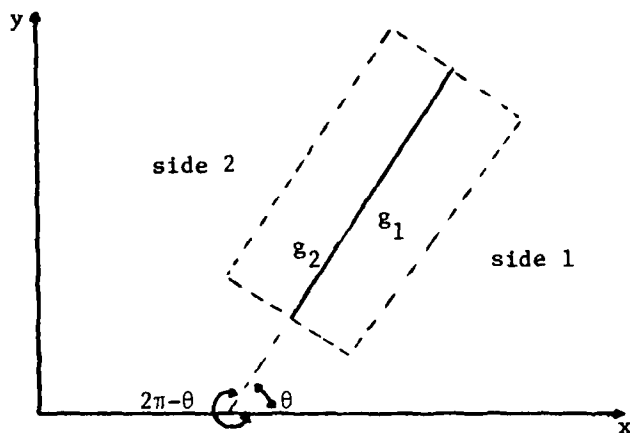
Figure 105.   Angle convention for g_1 and g_2.
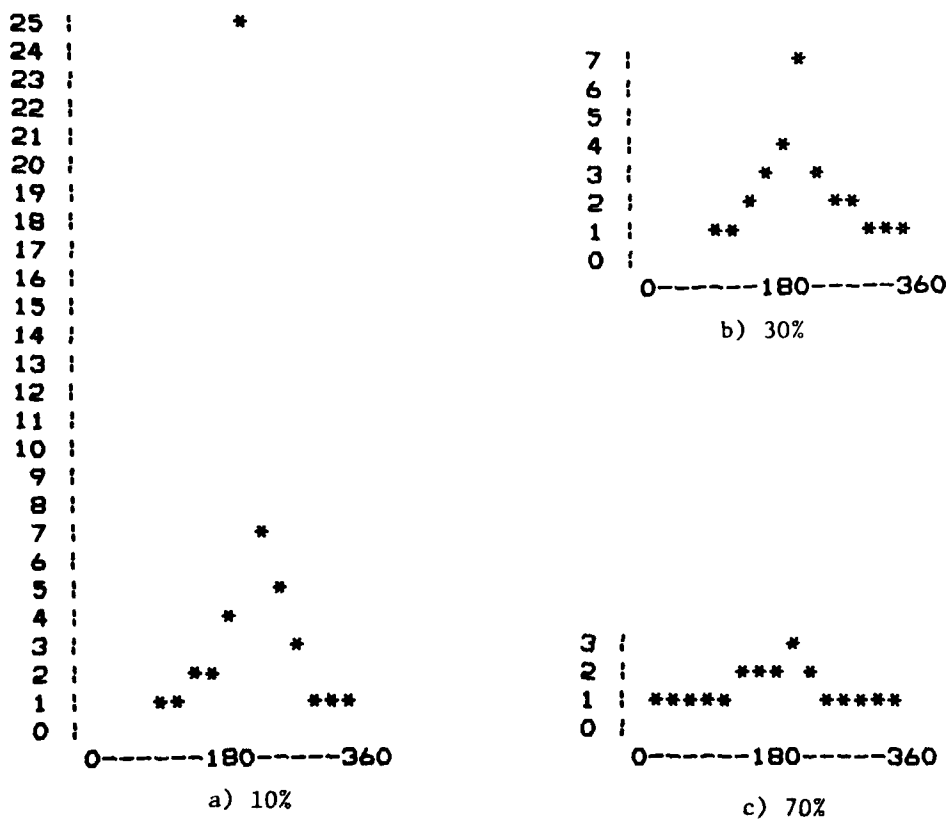
Figure 108. Plot of n_1/n_2 as a function of θ, after noise cleaning.

a) 10%

b) 30%

c) 70%
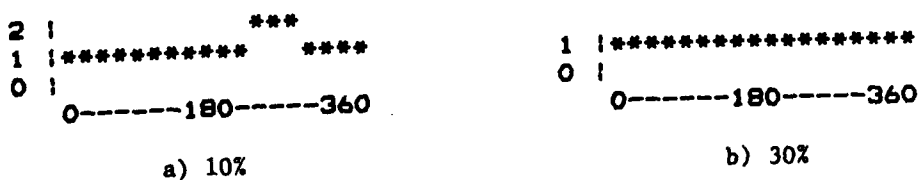
Figure 109. Plot of n_1/n_2 as a function of θ, before noise cleaning.

a) 10%

b) 30%
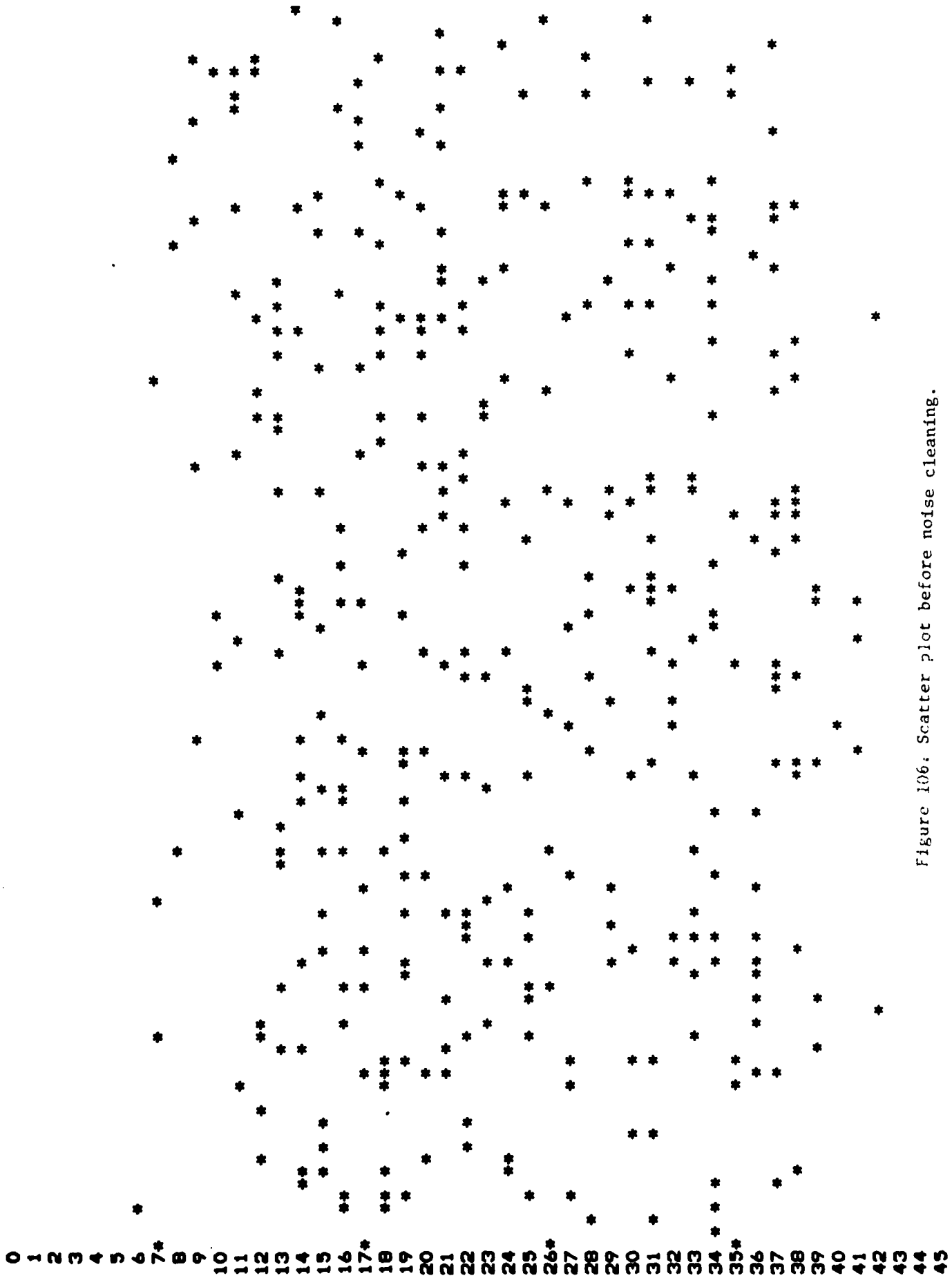
Figure 106. Scatter plot before noise cleaning.
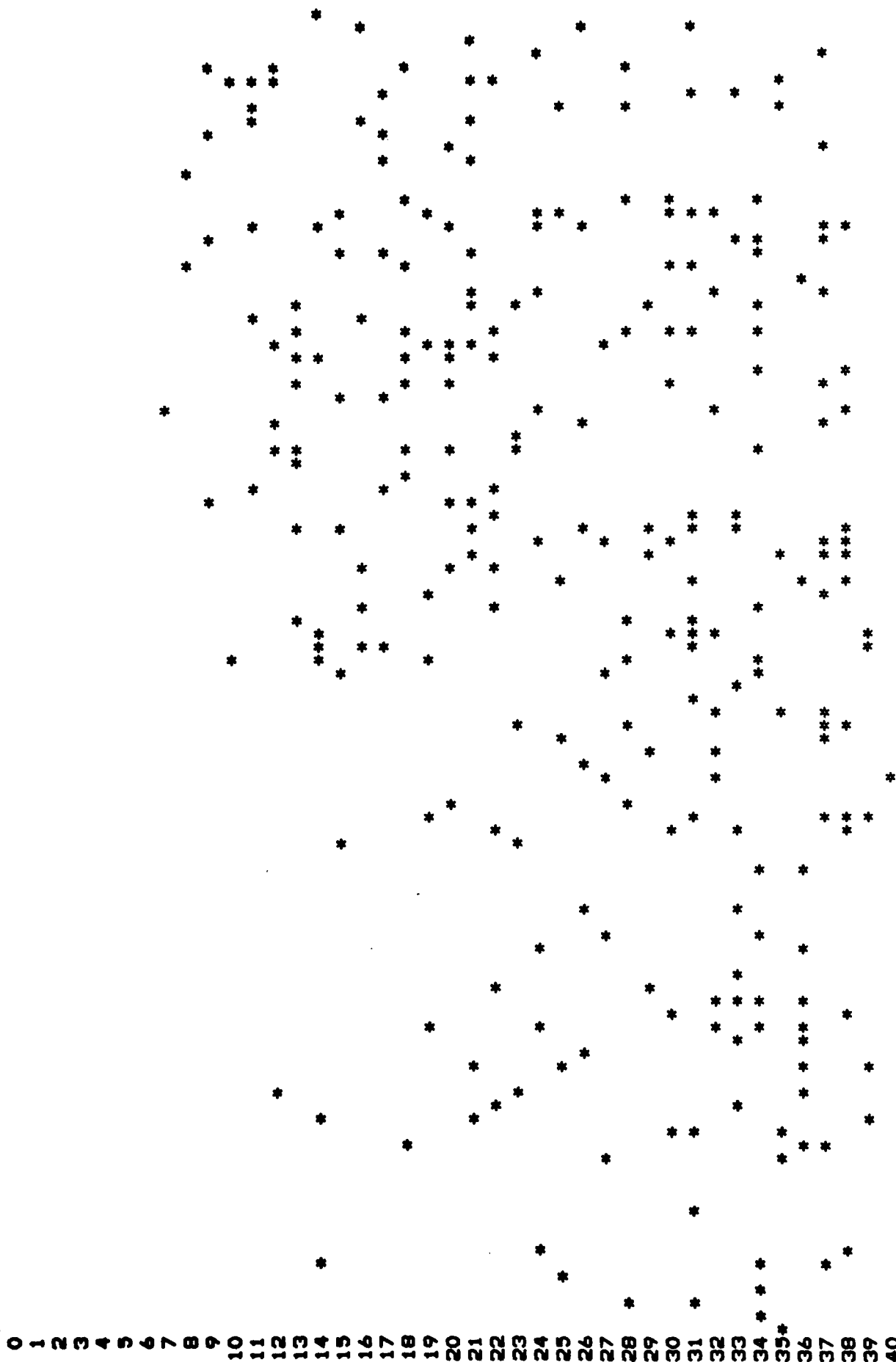
Figure 107. Scatter plot after noise cleaning.

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Buildings
Roads

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper describes steps toward the recognition of cultural features such as buildings and roads on aerial photographs. The approach involves several successive stages of grouping of edge segments. Straight line segments are fitted to sets of edge pixels; compatibilities between pairs of these segments, based on gray level and geometric information, are computed; and the segments are then grouped into building-like and road-like groupings based on these compatibilities. Examples of the results obtained using this

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approach are given, and some variations on the initial stages
of the process are also investigated.